Index

1-localic reflection, 358
∞-category, 335
  compact object, 372
  compactly generated, 362
  derived, 171, 180
  kernels and cokernels, 166
  presentable, 336, 349
  prestable, 170
  sheaves valued in, 350
  stable, 165, 365
∞-cosmos, 64
∞-groupoid
  presheaf of, 348
  sheaf of, 348
∞-topos, 348, 349
  colimits, 352
  descent, 353
  étale morphisms, 364
  internal hom, 352
  limits and colimits, 361
  point of, 373
  quasicompact, 397
  sheaves on, 350
  slices, 351
  spectrally ringed, 369
  terminal, 359
  truncation, 354
A_∞-algebra, 280
A_∞ operad, 192
A_∞-ring spectrum, 258, 284
abelian group scheme, 399
adic
  $\mathbb{E}_\infty$-ring, 388
  sheaf, 389
affine space
  over an $\mathbb{E}_\infty$-ring, 383
  smooth, 384
Algebraic Creation Property, 97
Algebraic Invariance Property, 98
André–Quillen homology, 282
  topological, 283
associative algebra operad, 192
Atiyah duality, 161
Azumaya algebra spectrum, 297
Barratt–Eccles operad, 193, 256, 267
bipermutative category, 271
Bott map, 394
Bousfield localization, 117, 254
  of a model category, 331
  of combinatorial model categories, 334
  of presentable ∞-categories, 336
  of spaces, 314
  of stable categories, 320
Bousfield–Friedlander spectra, 118
braided operad, 197
Brauer group
  derived, 298
  of a commutative ring spectrum, 297
brave new rings, 249
$\mathcal{C}_n$-space, 236
  group completion, 237
  grouplike, 236
category
  stable, 318
category of fractions, 8
Čech nerve, 355
closed model category, 13
closed symmetric monoidal category, 94

coalgebra
  commutative, 391
  smooth, 391
  smooth commutative, 391
cobordism, 161
cofibrant object, 17
cofibrant replacement, 17
cofibrantly generated model category, 221
commutative algebra operad, 191
commutative ring spectra, 249
commutative ring spectrum, 149, 252, 366
complete Segal space, 62
coordinate-free spectrum, 83, 115
cosmological functor, 66
cospectrum, 392
cylinder object, 18

Day convolution, 162, 265
degeneracy quotient object, 218
degeneracy subobject, 218
Deligne–Mumford stack, 345, 379
  finite limits of, 378
  flat map, 382
  formal spectral, 390
  morphisms of spectral, 375
  nonconnective spectral, 374
  quasiaffine nonconnective spectral, 397
  spectral, 374
  underlying, 379
derivation, 281
derivative, 158
derived functor
  left, 27
  right, 27
  total, 27
diagram categories, 109
DK-equivalence, 50
Dold-Kan correspondence, 151
double category, 31
dualizing line
  of a formal group, 393
Dyer–Lashof operations, 243

E-(co)homology, 157
E_∞-algebra, 260
E_∞ operad, 193, 196
$\mathbb{E}_\infty$-ring, 348, 366
  adic, 388
  étale map of, 370
  étale site of, 372
  finite characteristic, 368
  free, 368
  polynomial ring, 368
E_∞-ring spectrum, 241, 256, 284
E_n operad, 196
E_n space, 236
effective epimorphism, 355
Eilenberg–Mac Lane object, 357
Eilenberg–Mac Lane spectrum, 258, 272
Eilenberg-Mac Lane spectrum, 157
EKMM spectra, 77, 140, 253
elliptic cohomology, 345, 398
elliptic curve, 345, 401
  moduli stack, 398
  oriented, 402
elliptic spectrum, 346
endomorphism operad, 191, 339
enveloping algebra, 206, 229
enveloping operad, 212
étale cover, 349, 372
étale descent, 292
étale map, 290, 370
  of ∞-topoi, 364
  of $\mathbb{E}_\infty$-rings, 370
  of spectrally ringed ∞-topoi, 380
étale sheaf, 349
étale site, 399
  of an $\mathbb{E}_\infty$-ring, 372
  sheaves on, 372
étale spectrum
  of an $\mathbb{E}_\infty$-ring, 372

fibrant object, 17
fibrant replacement, 17
flat descent
  for $\mathbb{E}_\infty$-rings, 372
flat map
  of a nonconnective spectral DM stack, 382

formal completion
  of a spectral Deligne–Mumford stack, 390
formal group, 346
  over an $\mathbb{E}_\infty$-ring, 391
  Quillen, 392
formal hyperplane, 391
formal spectrum
  of an adic $\mathbb{E}_\infty$-ring, 389
function spectrum, 164
functor
  accessible, 349
  excisive, 153
  linear, 153
  of finite presentation, 153
  reduced, 153
functor of points
  of a nonconnective spectral Deligne–Mumford stack, 385
functorial factorization, 15

Galois descent, 292
Galois extension
  faithful, 288
  of commutative ring spectra, 288
Γ-space, 77, 146
Γ-space, 254
generalized cohomology theories, 345
geometric morphism, 359
geometric realization, 24, 47, 218
Grothendieck topology, 350, 358
groupoid core, 9

hammock localization, 50, 116
homology localization
  of spaces, 323
  of spectra, 328
homology operations, 256
homotopical category, 5, 11
homotopical functor, 26
homotopy category, 22, 39
  of a quasi-category, 56
  p-adic, 242
  rational, 242
homotopy coherent nerve, 58
homotopy coherent realization, 58
homotopy colimit, 41, 265
homotopy dimension, 358
  locally, 358
homotopy limit, 41
homotopy mapping space, 116
homotopy product, 39
homotopy sheaf, 356
homotopy theory
  p-adic, 241
  rational, 240
hypercomplete object in an ∞-topos, 357

I-spaces, 265
ideal of definition, 388
identity operad, 191
isofibrations, 64
iterated double mapping cylinder, 312, 332
iterated loop space, 236

J homomorphism, 160

Kan complex, 24
Kan extension
  absolute, 26
  left, 26
  right, 26
Kan fibrations, 24
Ken Brown's lemma, 29

latching object, 218
left deformation, 28
left homotopy, 18
Leibniz cotensor, 34
Leibniz tensor, 33
lifting criteria
  for localizations, 311
lifting property
  left, 12
  right, 12
linear isometries operad, 142, 193, 256, 260
little ∞-cubes operad, 195
little n-cubes operad, 194, 236, 256
local morphisms, 377
local sheaf, 376

locale, 361
localization
  categorical, 304
  smashing, 321
Loday construction, 275
logarithmic ring spectra, 269

map of operads, 191
  derived monad equivalence, 225
marked simplicial set, 63
matching object, 218
May universe, 83
minimal model, 244
model category, 13
  closed, 13
  cofibrantly generated, 221, 333
  combinatorial, 42, 93, 333
  monoidal, 35, 94, 340
  of algebra spectra, 259
  of differential graded algebras, 259
  of module spectra, 258
  of unbounded chain complexes, 258
  simplicial, 36
  stable, 87
  symmetric monoidal, 340
model structure
  for operadic algebras, 220
  injective, 42, 110
  Joyal, 60
  p-adic, 242
  positive, 254
  projective, 42, 109
  rational, 242
  Reedy, 45
  right induced, 253
module spectra, 366
  flat, 367
monad, 202
monomorphism in an ∞-category, 355
Moore algebra, 229, 233
Moore loop space, 233
Morava K-theory, 107
multicategory, 342

n-connective
  in an ∞-category, 354
non-symmetric operad, 192
nullification, 305

obstruction theory, 284
operad, 189
  A_∞, 192
  associated to a non-symmetric operad, 192
  associative algebra, 192
  Barratt–Eccles, 193, 256, 267
  braided, 197
  change of operad adjunction, 224
  commutative algebra, 191
  E_∞, 193, 196
  E_n, 196
  endomorphism, 191
  enveloping, 212
  identity, 191
  linear isometries, 193, 256
  little ∞-cubes, 195
  little n-cubes, 194, 236, 256
  map of, 191
  monad associated to, 202
  non-symmetric, 192
  tensor product of, 197
operadic algebra, 199, 201
  colimits, 210
  comparison theorems, 224
  limits, 210
  model structure, 220
  module over, 204
  rectification theorems, 224
orthogonal spectrum, 76, 129

p-adic equivalence, 322
p-complete spectrum, 322
p-completion, 322
p-adic homotopy theory, 241, 242
p-adic model structure, 242
path object, 18
permutative category, 270
Picard group
  algebraic, 294
  of a commutative ring spectrum, 294

of a symmetric monoidal category, 293
Postnikov towers, 286
power operations, 256
preorientation
of a formal group, 393
prespectrum, 141
projective space
over an $\mathbb{E}_\infty$-ring, 384
smooth, 385
Pushout-Product Axiom, 95

quasi-category, 56
quasicoherent sheaves, 395
descent, 396
pullback, 396
pushforward, 396
Quillen adjunction, 30
Quillen equivalence, 37
Quillen functor
left, 29
right, 29
Quillen two-variable adjunction, 35

rational homotopy theory, 240, 242
rational model structure, 242
Reedy category, 44
Reedy weak factorization system, 45
relative category, 63
right homotopy, 18
ring spectrum
A_∞, 258, 284
E_∞, 241, 256, 284

S-equivalence, 305, 308
fiberwise, 316
S-local, 305, 308
S-localization, 305, 309
fiberwise, 316
schematic objects, 380
Segal category, 62
Serre classes of abelian groups, 301
simplicial commutative ring, 369
simplicial set, 24
small object argument, 314
smash product of spectra, 125, 163
smooth
coalgebra, 391
sober space, 360
solution sheaf, 375
Spanier-Whitehead duals, 159
spectral algebraic geometry, 346
spectral scheme, 375
nonconnective, 374
spectrally ringed ∞-topos, 369
affine, 374
étale map of, 380
sheaves of modules, 395
spectrum, 153, 365
p-complete, 322
connective, 176, 365
dual, 164
in an ∞-category, 174
n-connective, 365
n-truncated, 365
sequential, 155
suspension, 159
stable spherical fibration, 161
Steenrod operations, 243
strictly Henselian
sheaf of $\mathbb{E}_\infty$-rings, 376
symmetric sequence, 120
symmetric spectra, 253
symmetric spectrum, 76, 122
semistable, 266

t-structure, 175
complete, 179
separated, 179
Thom spectra, 161, 260
graded Thom spectra, 269
topological Hochschild homology of, 278
via $\mathbb{L}$-spaces, 261
via I-spaces, 265
topological André–Quillen homology, 283
TAQ-étale, 290
topological Hochschild homology, 273
of Thom spectra, 278
THH-étale, 290
topological modular forms, 295, 346, 400

totalization, 47
two-variable adjunction, 33
units of ring spectra, 260
  graded units, 267
vanishing locus
  of an adic $\mathbb{E}_\infty$-ring, 388
variety
  abelian, 401
  over an $\mathbb{E}_\infty$-ring, 400
  strict abelian, 401
$\mathcal{W}$-space, 77
$\mathcal{W}$-spectrum, 146
weak factorization system, 12
weighted colimit, 45
weighted limit, 45
Yoneda lemma, 219
Zariski cover, 373
Zariski spectrum
  of an $\mathbb{E}_\infty$-ring, 373