This comprehensive text focuses on the homotopical technology in use at the forefront of modern algebraic topology. Following on from a standard introductory algebraic topology sequence, it will provide students with a comprehensive background in spectra and structured ring spectra. Each chapter is an extended tutorial by a leader in the field, offering the first really accessible treatment of the modern construction of the stable category in terms of both model categories of point-set diagram spectra and infinity-categories. It is one of the only textbook sources for operadic algebras, structured ring spectra, and Bousfield localization, which are now basic techniques in the field, and the book provides a rare expository treatment of spectral algebraic geometry. Together the contributors Emily Riehl, Daniel Dugger, Clark Barwick, Michael A. Mandell, Birgit Richter, Tyler Lawson, and Charles Rezk offer a complete, authoritative source to learn the foundations of this vibrant area.



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# Stable Categories and Structured Ring Spectra

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