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Evaluating territories of Go positions with capturing races

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In analysing capturing races, or *semeai*s, we have been focusing on the method to find which player wins the race so far, because whether to win or to lose the capturing race largely affects the territory score and it somtimes can decide the outcome of the game. But in order to evaluate the state of the game properly, we usually have to count the territory score precisely regardless of which player wins the race. Sometimes the loser of a capturing race has good moves although the moves don't affect the status of winning or losing the race. In this paper, we propose a method for evaluating territory score in each decomposed subgame of a capturing race considering the status of the winner of the race.

1. Introduction

Combinatorial game theory has been applied to many kinds of existing games and has produced many excellent results. In the case of the game of Go, applications of CGT have been focused on endgames [Berlekamp and Wolfe 1994; Berlekamp 1996; Müller et al. 1996; Nakamura and Berlekamp 2003; Spight 2003] and eyespace values [Landman 1996] so far. But it can be applied to any situations that involve counting. Recently, we developed a new genre of application of CGT to Go, that is, to count liberties in capturing races [Nakamura 2003; Nakamura 2009; Nakamura 2006].

Capturing races, or *semeai* is a particular kind of life and death problem in which two adjacent opposing groups are each fighting to capture the opponent's group. A player's strength in Go depends on their skills in winning capturing races as well as opening and endgame skills. In order to win a complicated capturing race, various techniques in counting liberties, taking away the opponent's liberties, and extending self-liberties, are required in addition to broad and deep reading. Human expert players usually count liberties for each part of the blocks involved in semeai, sum them, and decide the outcome. A position of capturing races can also be decomposed into independent subpositions, as in the cases of endgames and eyespaces, and we can apply CGT to analyse the capturing races. We propose

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a method of analysing capturing races that have no shared liberty or have only simple shared liberties, and then, using combinatorial game values of external liberties, give an evaluation formula to find the outcome of the capturing races.

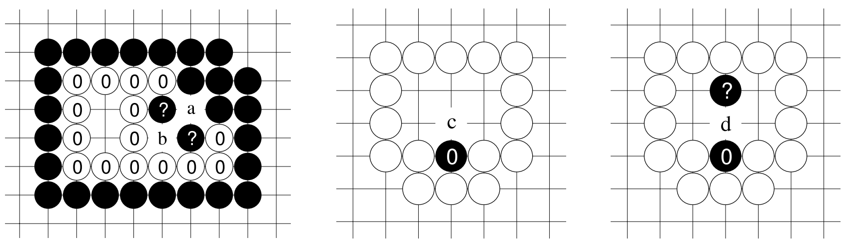
Past methods of analysing capturing races have focused on ways to find which player wins the race, because winning or losing the capturing race largely affects the territory score and sometimes it can decide the outcome of the game. But in order to evaluate the state of the game properly, we usually have to count the territory score precisely regardless of which player wins the race. Sometimes the loser of a capturing race has good moves although the moves don't affect the status of winning or losing the race. In this paper, we propose a method for evaluating territory score in each decomposed subgame of a capturing race that takes into account the status of the winner of the race.

2. Analysing capturing races using CGT

2.1. ***How to decide the winner.*** In order to model capturing races, we define the Liberty Counting Game (LCG), which has the same rules as Go except for scoring. I briefly explain LCG below. More details can be found in [Nakamura 2003; Nakamura 2009; Nakamura 2006].

In LCG, the terminal score is basically the number of liberties of essential blocks,[1] but it is exactly the number of opponent's moves that are required to take away all the liberties of essential blocks. By convention, Black is Left and White is Right, Black scores are positive and White scores are negative.

Figure 1 shows some examples of CGT values of LCGs. In part (a), White's essential block[2] has three liberties, but Black cannot directly attack White's external liberty *b*, because if he simply fills the liberty *b*, Black's attacking block gets to be



(a) -4 (b) $\{3 \mid 0\}$ (c) $\{4 \mid 0\}$

Figure 1. CGT values of LCGs.

[1] A block of connected stones involved in a capturing race is called an essential block, if capturing the block immediately decides the race.

[2] The block of circled stone denotes an essential block. In this example, the circled block is not involved in semeai, but in LCG we just count liberties of essential blocks.

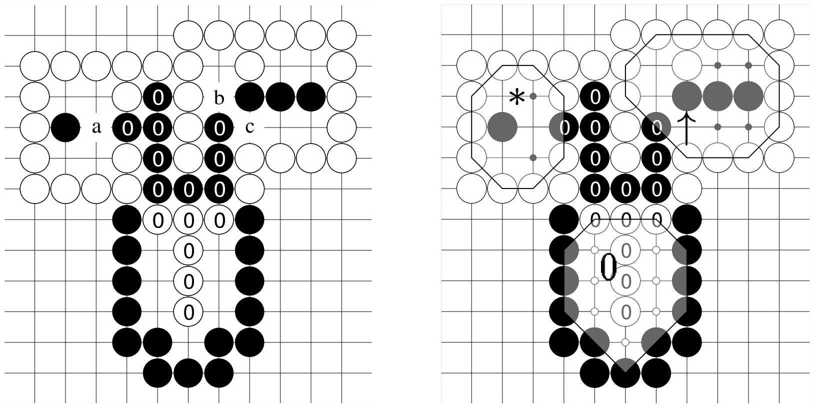


Figure 2. Left: an example problem. Right: its analysis using CGT.

in *atari*, and White can capture Black's three stones by playing to *a* and White's essential block becomes alive. So, Black needs to spend one move to protect *a* prior to attack. Generally, liberty scores in external liberty regions are greater than or equal to the number of liberties of essential blocks. In part (b) of the figure, if White plays first, the score is zero, but if Black plays to *c* first, the number of liberties becomes 3. In (c), Black can connect his two stones of an essential block and a neutral block playing to *d* and the score becomes 4, if he plays first.

Figure 2, left, is an example semeai problem. The right side of the figure shows an analysis of each subgame. The upper left subgame is $\{4 \mid 0\}$, the upper right subgame is $\{6 \mid \{4 \mid 0\}\}$ and the lower subgame is -7. We cool these subgames by two degrees and obtain $2*$, $4\uparrow$ and -7, respectively. The total value is $-1\uparrow*$ and it is incomparable to -1. If Black plays first, he can round the value up to 0 and wins the race by one move. If White plays first, he can round the value down to -2 and wins the race by three moves. So the first player wins the race of Figure 2. The Black's only winning move is move *a*. After a sequence of Black *a*, White *b* and Black *c*, the number of liberties of Black's essential block is 8 and the number of liberties of White's essential block is 7 and Black definitely wins the race by one move. On the other hand, White has two winning moves of move *a* and move *b*. After White *a*, even if Black moves to *b*, White can win the race by two moves.[3] Alternatively, after White *b* for his first move, move *a* and move *c* are *miai* and White can win the race by three moves. In consequence, White *a* loses one liberty count in semeai compared to White *b*. But this loss is not only in liberty count but also in territory score. We will discuss the territory score involved in semeais in the next section.

[3] We assume the winner should play the last move in showing how many moves ahead, even if he doesn't need to play any moves to win. In this case, we take a White's extra move into account.

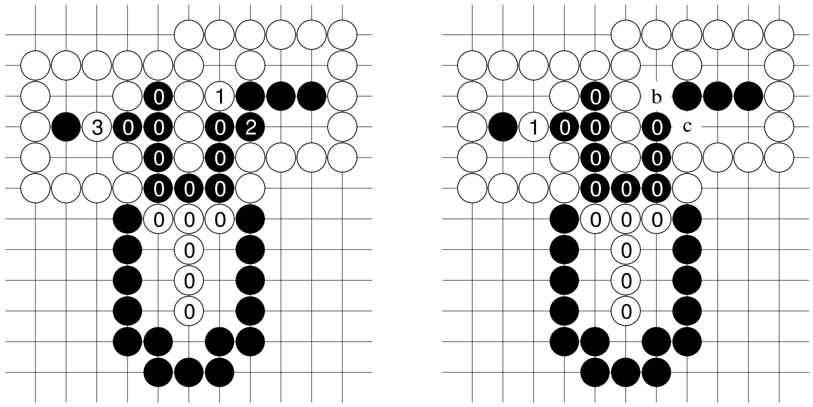


Figure 3. Two winning sequences for white.

2.2. ***Territory scores.*** Figure 3, left, shows a winning sequence for White after White *b* in 2.

Black's 13 stones of the essential block are bound to be captured eventually. White has six territory points in the upper left subregion, one point in the upper right subregion and 26 points as capturing Black's 13 stones. Black has no territory points because all the empty points in the lower subregion are *dame*. So the total is 33 points for White. In Figure 2, right, on the other hand, if White *b* and Black *c* are played after White 1, the resulting position is identical to the position of the left part of the figure and the total is also 33 points. But if Black plays the move *b*, Black can reduce White's territory points although the move doesn't change the status of the capturing race. After Black *b*, White's territory score is 32 $(= 6+26)$ points. So Black's move *b* takes away one White's territory point compared with the case of White *b*.[4] If White plays both the move *b* and the move *c* in Figure 3, right, the total score increases to 35 points.[5]

Generally, this kind of phenomena in capturing races is called *semedori*. As in the above analysis in terms of territory score, if we search all the possible winning sequences of an entire capturing race after we analyse the status of the capturing race, we can figure out the final territory score. But the procedure is not efficient because of combinatorial explosion. In the next section, we will show a method to evaluate territory score on each subgame and to combine it taking into account the status of capturing races and semedori.

[4]This reduction is also against Figure 3, left.

[5]White's moves of *b* and *c* cut off Black's three stones in the upper right region and the territory score of the upper right subregion becomes $1 + 10 = 11$. Since the number of stones of Black's essential block is 9, the total score is $6 + 11 + 18 = 35$.

3. Evaluating territory scores of subgames involved in capturing races

For each subgame we introduce three auxiliary games in order to evaluate, without a full search, the territory score of a position involved in capturing races:

G_{lc}: The Liberty Counting Game (LCG).

G_{win}: A game whose score is partial territory score, assuming that the defender (player who owns the essential block in the region) is the winner of the entire capturing race.

G_{lose}: A game whose score is partial territory score, assuming that the defender (player who owns the essential block in the region) is the loser of the entire capturing race.

We use G_{lc} to decide the status of the capturing race. Either G_{win} or G_{lose} are used to evaluate the territory score according to the status of the capturing race.

Although each subgame should be independent in Combinatorial Game Theory, some subgames may depend on each other in the game of Go. Müller proposed a framework of Conditional Combinatorial Game (CCG) to describe games with some dependency [Müller 2003].

3.1. ***Conditional Combinatorial Game.*** The Conditional Combinatorial Game (CCG) is described as follows.

$$G \stackrel{\text{def}}{=} \left\{ L^1_{C_1},\ L^2_{C_2},\ L^3_{C_3},\ \ldots \;\middle|\; R^1_{D_1},\ R^2_{D_2},\ R^3_{D_3},\ \ldots \right\}.$$

Here L^i is a game to which Left moves from G and R^i is a game to which Right moves from G. Each C_i and D_i is some kind of predicate to check each move is legal or not using global information.

For our purposes, we can describe G_{win} and G_{lose} as a game of CCG using G_{lc} as the conditional predicates.

3.2. ***Territory scores of subgames.*** We show some subgames of capturing races and their corresponding games of G_{lc}, G_{win} and G_{lose} in Table 1.

The Territory score is the points of territory other than the essential block in each region. For example, G_{lose} of the subgame A is $\{-4 \mid -6\}$. We calculate the value as follows.

> If White cuts off Black's one stone, White's territory score becomes six points in the region. On the other hand, if Black connects the stone and the essential block, White has to play four more moves in this region in order to to capture Black's block. So, White's territory score becomes four points, that is, two captured stone, other than the essential block, and two territory points.

	liberty score		territory score			
	G_{lc}	cooled value	G_{win}	G_{lose}	cooled value	baseline
A	$4 \mid 0$	$2*$	$0 \mid -6$	$-4 \mid -6$	$-5*$	-7
B	$6 \Vert 4 \mid 0$	$4\uparrow$	$0 \Vert -1 \mid -11$	$-8 \Vert -9 \mid -11$	$-9\uparrow$	-13
C	$0 \mid -4$	$-2*$	$8 \mid 0$	$8 \mid 6$	$7*$	9
D	$0 \Vert -2 \mid -6$	$-2\uparrow$	$13 \Vert 6 \mid 0$	$13 \Vert 12 \mid 10$	$12\uparrow$	14
E	-7	-7	0	0	0	7
F	$-5 \mid -9$	$-7*$	$8 \mid 0$	$8 \mid 6$	$7*$	14
G	$-5 \mid -8$	$-6\frac{1}{2}$	$6 \mid 0$	$6 \mid 5$	$5\frac{1}{2}$	12
H	$8 \mid 4 \Vert 2 \mid\mid\mid 0$	$2\Downarrow*$	$0 \mid -6 \Vert -9 \mid\mid\mid -12$	$-8 \mid -10 \Vert -11 \mid\mid\mid -12$	$-11\Downarrow*$	-13
I	$6 \mid 3 \Vert 1$	$2\frac{3}{4}$	$0 \mid -1 \Vert -2 \mid -8 \mid\mid\mid -15$	$-13 \mid -14 \Vert -15$	$-14\frac{1}{4}$	-17
J	$3 \mid 0$	$1\frac{1}{2}$	$0 \mid -2 \Vert -5$	$-4 \mid -5$	$-4\frac{1}{2}$	-6

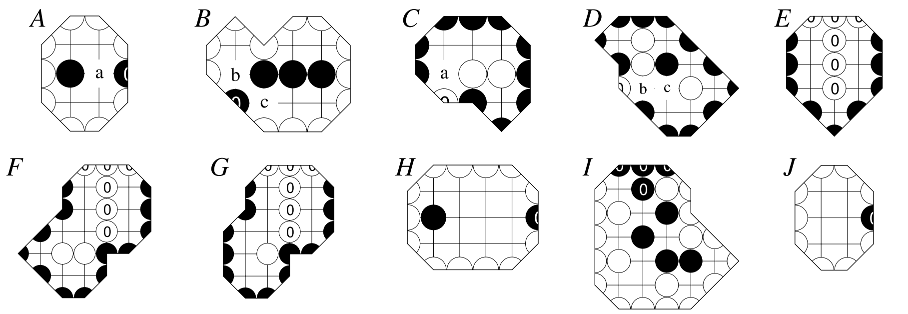


Table 1. Evaluation of liberty and territory for subgames A–J.

The “baseline” column means the case where we assume that the subregion is not involved in *semeai* and the opponent’s essential block is already dead. The territory score of the dead essential block is excluded from the baseline value.

A formula to calculate the cooled value of G_{lose} is

$$\text{Cool}(G_{\text{lose}}, 1) = \text{baseline value} + \text{Cool}(G_{\text{lc}}, 2), \qquad (1)$$

$\text{Cool}(G, t)$ being the game G cooled by t degrees. Formula (1) works as follows:

> The winner of capturing races has to fill all the liberties of the opponent’s essential block and capture it eventually. In case of *semedori*,

the liberty filling moves are played in his territory region. The number of liberty filling moves equals to the number of liberty count of the opponent's essential block in the subregion. The moves reduce his own territory points and we need to subtract the number from the baseline of the territory points. The winner's own territory region is the subregion of G_{lose} and the number of liberty count G_{lc} is the number of liberties of opponent's essential block, so positive and negative are reversed. Consequently we should add the baseline and the cooled value of G_{lc} in order to take into account the reduction.

We can evaluate the territory score taking into account the winner of the capturing race as follows.

Figure 2, left, is an example position of a capturing race combined A, B and E in Table 1. If White plays first, White can win the race and the total territory score is the sum of G_{lose} of A, G_{lose} of B and G_{win} of E. The cooled value of the total is

$$-5* + -9\uparrow + 0 = -14\uparrow*.$$

Considering the number of stones of each essential block, that is, Black's nine stones and White's six stones, we can figure out the final score is

$$-14\uparrow* - 18 = -32\uparrow*.$$

On the other hand, if Black plays first, the situation is more complicated. The total territory score is the sum of G_{win} of A, G_{win} of B and G_{lose} of E. White's moves in the region of A and B can be effective attacking moves to Black's essential block and the status of the capturing race can be changed. In that case, whether White's attacking moves and Black's responses are good or bad should be evaluated in terms of not only the gain in territory points in G_{win} but also the result of G_{lc} as the condition of CCG at the highest priority. As a result, White b is sente in terms of the status of the capturing race after Black's move a in Figure 2, so Black has to respond the move c immediately. In consequence, G_{win} of B becomes -1 from $\{0 \mid \{-1 \mid -11\}\}$ and the final score is -1.

If two subgames have the same property of G_{lc}, we can compare them in terms of territory. G_{lc} of the subgame A and G_{lc} of the subgame C have the same infinitesimal part of $*$. So, in case that Black is the winner, we compare G_{win} of A with G_{lose} of C and it will turn out that A is hotter than C. On the contrary, in case that White is the winner, we compare G_{lose} of A with G_{win} of C and it will turn out that C is hotter than A. We can also compare B with D because G_{lc} of B and G_{lc} of D have the same infinitesimal part of $\uparrow$. In case that Black is the winner, we compare G_{win} of B with G_{lose} of D and it will turn out that B is preferred to D. On the contrary, in case that White is the winner, we compare G_{lose} of B with G_{win} of D and it will turn out that D is hotter than B.

3.3. ***More examples.*** We can combine subgames A, B, C and D in Table 1, add some extra liberties and construct semeai problems that have same liberty counts. For example, all the semeai games of $A+B-7$, $A+D-1$, $C+B-3$ and $C+D+3$ have the same liberty count of $-1{\uparrow}{*}$, because

$$\begin{aligned}
&(1)\ A+B-7 &&= && 2{*} + 4{\uparrow} - 7 &&= && -1{\uparrow}{*},\\
&(2)\ A+D-1 &&= && 2{*} - 2{\uparrow} - 1 &&= && -1{\uparrow}{*},\\
&(3)\ C+B-3 &&= && -2{*} + 4{\uparrow} - 3 &&= && -1{\uparrow}{*},\\
&(4)\ C+D+3 &&= && -2{*} - 2{\uparrow} + 3 &&= && -1{\uparrow}{*}.
\end{aligned}$$

Although we can conclude that first player wins for all the semeai games, it is not easy to figure out the optimal path in terms of territory score for each of the semeai games from (1) to (4). Figure 4 shows the canonical game tree of ${\uparrow}{*}$. But it's not true in semeai games of the sum of G_{lc}s. It's enough for the attacker to win the race, because whether to win or to lose usually makes a big difference in territory scores and he doesn't need to gain extra liberties at the risk of losing territory scores. Canonicalization process for semeai games may prune some effective branch in terms of territory score.

Figure 5 shows the game tree of the semeai games from (1) to (4). Heavy lines denote winning paths in semeais for each player and numbers at the leaf nodes are the liberty score. Boxed numbers under the leaf nodes denote the territory score for each of semeai games and italic numbers in heavy boxes mean the nodes should be selected in terms of territory score by priority. For example, in semeai game (1), if Black plays first, Black should play move a. At this point, White b is sente and Black has to respond move c immediately. The territory score becomes -1 as described in Section 3.2. But if White plays first, White b

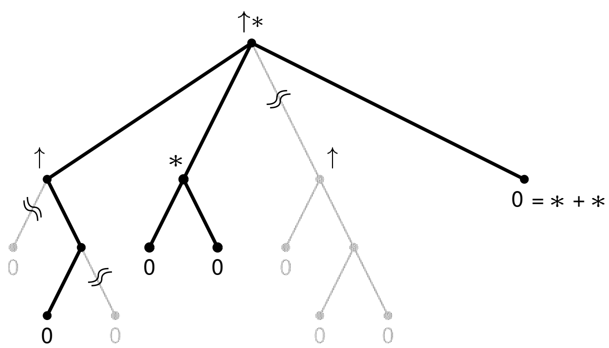


Figure 4. Canonical game tree of ${\uparrow}{*}$.

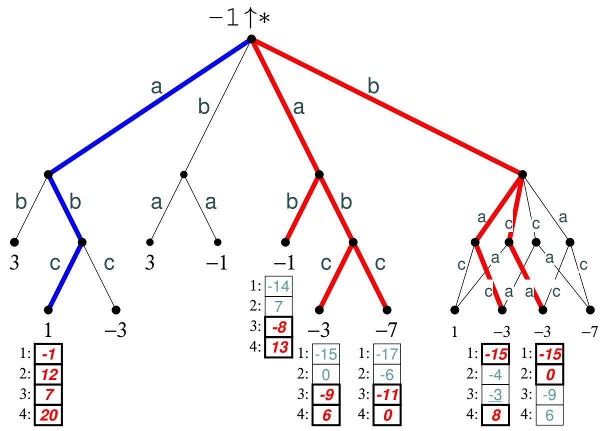


Figure 5. Game tree of $-1{\uparrow}{*}$ semeai.

will be preferred to a. Then a and c are miai and the score will be -15. In case of the semeai game (3), White's optimal move is different from the above case. White should play move a and the chilled territory score becomes $-9{\uparrow}$.

If we give some more liberties for Black, the situation will be changed as shown in Figures 6 and 7.

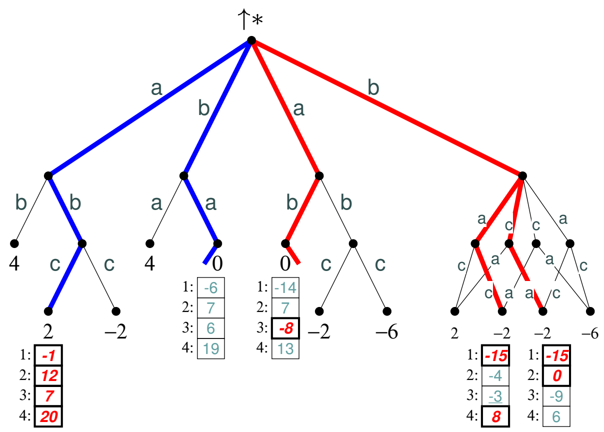


Figure 6. Game tree of ${\uparrow}{*}$ semeai.

4. Summary

We proposed a method for evaluating territory score in each decomposed subgame of a capturing race considering the status of the winner of the race. We introduced three different kinds of games, $G_{\rm lc}$, $G_{\rm lose}$ and $G_{\rm win}$, for each subgame and showed

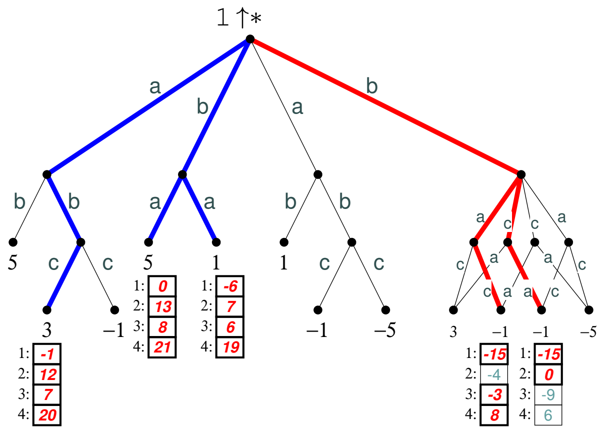


Figure 7. Game tree of 1↑∗ semeai.

a method to evaluate territory score of a position involved in capturing races without entire search.

G_{lose} can be calculated from G_{lc} using the formula (1), but it is difficult to evaluate G_{win} in combination with other subgames because the canonicalization process for G_{lc} may prune some effective branch in terms of territory score. So future work includes how to select good moves in G_{win} for each players efficiently keeping the status of the entire semeais.

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