

Symplectic geometry has its origin in physics, but has flourished as an independent subject in mathematics, together with its offspring, symplectic topology. Symplectic methods have even been applied back to mathematical physics; for example, Floer theory has contributed new insights to quantum field theory. In a related direction, noncommutative geometry has developed an alternative mathematical quantization scheme, based on a geometric approach to operator algebras. Deformation quantization, a blend of symplectic methods and noncommutative geometry, approaches quantum mechanics from a more algebraic viewpoint, as it addresses quantization as a deformation of Poisson structures.

This volume contains seven articles based on lectures given by invited speakers at two May 2010 workshops held at MSRI: *Symplectic and Poisson Geometry in Interaction with Analysis, Algebra and Topology* (honoring Alan Weinstein, one of the key figures in the field) and the *Hayashibara Forum on Symplectic Geometry, Noncommutative Geometry and Physics*. Both workshops were jointly sponsored by MSRI, the Research Institute of Mathematical Sciences at Kyoto University (RIMS), and the Hayashibara Foundation. The articles include presentations of previously unpublished results and comprehensive reviews including recent developments in these areas.



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Symplectic, Poisson, and Noncommutative Geometry

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# Symplectic, Poisson, and Noncommutative Geometry

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