

This is a collection of surveys and primarily expository articles focusing on recent developments concerning various quantitative aspects of “thin groups.” There are discrete subgroups of semisimple Lie groups that are both big (Zariski dense) and small (of infinite covolume). This dual nature leads to many intricate questions. Over the past few years, many new ideas and techniques, arising in particular from arithmetic combinatorics, have been involved in the study of such groups, leading, for instance, to far-reaching generalizations of the strong approximation theorem in which congruence quotients are shown to exhibit a spectral gap, referred to as superstrong approximation. This book provides a broad panorama of a very active field of mathematics at the boundary between geometry, dynamical systems, number theory, and combinatorics. It arose from the MSRI Hot Topics workshop of the same name in February 2012.

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Thin Groups and Superstrong Approximation

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