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Chapter 19
English Learners and Mathematics Learning: Language Issues to Consider

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The Problem

TUTOR: "OK — let's begin with some of the terms you are going to need for this kind of math."
STUDENT: "Whadyu mean 'terms'?"
TUTOR: "You don't know the word, 'term'?"
STUDENT: "Nuh-uh."
TUTOR: "Have you ever heard it before?"
STUDENT: "No — huh-uh."

An unpromising start for the tutoring session: the student, a "former" English learner, needs help with math to pass the California High School Exit Examination (CAHSEE). He has failed the math part of the test twice already, and will not graduate from high school if he does not improve his performance substantially this year.[1] But as we see in the vignette, not only does this student have to learn more math this year — he also has to gain a much better understanding of the language used in math.

Although "Freddy" is no longer categorized as an English learner at school, his English remains quite limited. When he began school 12 years earlier, he was a Spanish monolingual. Now, at age 17, he speaks English mostly, although Spanish is still spoken in the home. Despite having been in a program described

[1] A summary of the results for the 2004 administration of the CAHSEE [California 2004b] indicates that of the nearly half million tenth-grade students who took the test in 2004, 74% passed the math part of the test. English-language learners performed 25% lower, with a 49% passing rate. In the English language arts part of the test, 75% of all students passed, while the passage rate for English-language learners was 39%, or 36% lower than all students.

as "bilingual" during his elementary school years, his schooling has been entirely in English from the time he entered school. The "bilingual" program provided him and his Spanish-speaking classmates one Spanish language session each week. Over the years, he has learned enough English to qualify as a "fluent English-speaker."[2] His English is adequate for social purposes, allowing him to communicate easily enough with peers and teachers. However, he lacks the vocabulary and grammatical resources to make much sense of the materials he reads in school, or to participate in the instructional discourse that takes place in his classes. Like many former English learners in California, Freddy has language problems at school, but he is no longer included in the official record of English learners kept by the California State Department of Education.

Officially, there are 1.6 million English learners in California's public schools. They constitute 25% of the student population, grades K through 12, for whom language is a clear barrier to the school's curriculum. Unofficially, there are many others—students like Freddy—who have learned enough English through schooling to be classified as English speakers but whose problems in school nonetheless can be traced to language difficulties. Added to the students who are still classified as English learners, they comprise some 2.25 million students in the state whose progress in school is impeded because they do not fully understand the language in which instruction is presented.

Would Freddy have been helped if this session had been conducted in Spanish, his primary language? The answer is no — not anymore. It would have helped when he was younger, but now, at age 17, it makes no difference. Having been schooled exclusively in English, Freddy no longer understands or speaks Spanish as well as he does English. He was no more familiar with Spanish '*término,*' than he was with English 'term.'

Had he heard of expressions that included the word, 'term' — for example, "term paper," "terms of endearment," and "term limit"?

"Well, no, not really."

How important is the word 'term' itself? It was easy enough to tell him that in this context, 'term' means 'word' — words that will be used in discussing the mathematical concepts and operations that he would be learning about. His lack of familiarity with such a word, however, is telling. He knows and can use everyday words, but he lacks familiarity with words that figure in academic discourse — the kind of language used in discussing academic subjects such as mathematics, science, and history. But the requirements of language go beyond vocabulary; they include familiarity with distinctive grammatical structures and

[2] English learners are designated "fluent English proficient" when they achieve a certain score on a test of English proficiency administered by their school districts. The test California schools must use for this purpose is the California English Language Development Test.

rhetorical devices that add up to a different register of the language that is used in academic settings. Clearly, Freddy needed help with academic English as much as he did with the mathematics concepts he needed to learn. The problem, however, is that while he recognized and acknowledged his need for help with math ("Me and math jus' don't get along"), he was quite oblivious to his need for help in English. In this, he is not unlike the many mathematics teachers he has had over the years. He was asked:

> "Math makes use of many special terms, such as *probability, radius,* and *factor*. Did your teachers explain what those terms meant?"
>
> "Well, yeah — but I still don't know what they mean."

Evidently, the explanations that teachers have offered have not been meaningful or memorable enough, because Freddy could not say what any of these terms meant, nor would he venture any guesses. It was not surprising then that he would have difficulty dealing with the CAHSEE *Mathematics Study Guide* that the California State Department of Education makes available to students preparing to take the test. The Guide offers Spanish equivalents for key terms in the expectation that students whose primary language is Spanish would find it easier to understand the concepts if they were identified in Spanish. Consider for example, the following definitional paragraph from the glossary of the 2004 CAHSEE *Study Guide* [California 2004a, p. 83]:

> The **probability** of an event's happening is a number from 0 to 1, which measures the chance of that event happening. The probability of most events is a value between 0 (impossible) and 1 (certain). A probability can be written as a fraction, as a decimal or as a percentage. Spanish word with the same meaning as *probability:* probabilidad.

Language in Learning

Freddy could read the passage, but did he understand it? What does it mean to say, "the probability of most events is a value between 0 (impossible) and 1 (certain)"?[3] What could it possibly mean to say, "a probability can be written as a fraction, as a decimal or as a percentage"? His efforts to paraphrase what he had read revealed that he understood virtually nothing of this definition, nor was

[3] The adequacy of the definitions provided in the *Study Guide* is not an issue in this discussion — nevertheless, it should be noted that this is not the clearest explanation of the concepts of probability, or of dependent and independent events! For example, it does not make any appeal to the ordinary use of "probable," "probably," or "probability" in terms of chance or likelihood. The technical term is defined as a number, or a numerical value, which is correct, but except for the phrase that it "measures the chance" there is no exploitation of the everyday concept.

he helped by the inclusion of the Spanish equivalent, *probabilidad*, a word that was entirely unfamiliar to him. ‘*Probabilidad*’ is as much a technical term in Spanish as ‘probability’ is in English. His teachers used some Spanish in school through the second grade, but mathematics, even then, was taught in English. There was little likelihood that he would have learned mathematical terminology in Spanish even in school. He was asked to read on, in the hope that this text from the *Study Guide*’s glossary would help clarify the concept of probability.

> **Independent events, dependent events:** These terms are used when figuring probabilities. In probability, an event is a particular happening that may or may not occur. Some examples of events are: “A fair coin will come up heads on the next flip,” and “Rain will fall in Oakland tomorrow,” and “Trudy Trimble will win next week’s California lottery.”
>
> One event is said to be independent of another if the first event can occur with absolutely no effect on the probability of the second event’s happening. For example, suppose you are going to flip a fair coin two times and on the first flip it comes up heads. On the second flip, the probability of the coin coming up heads is still 50%. Each flip of the coin is *independent* of all other flips.
>
> But some events are *dependent*; that is, the probability of one event depends on whether the other event occurs. For example, suppose you are randomly choosing two marbles, one after another, from a bag that contains three blue marbles and three red marbles. On your first draw, you have a 50% chance of drawing a blue marble. But on your second draw, the probability of drawing a blue marble depends on which color you pulled out on the first draw. The probability of getting a blue marble on your second draw is *dependent* upon the result of the first draw. Spanish words with the same meaning as *independent events, dependent events*: eventos independientes, eventos dependientes. [California 2004a, p. 81]

When Freddy was done reading this passage, he was even less sure that he understood what “probability” meant than he was before reading it. Added to his confusion was what this word had to do with coins, fair or not, or with rain in Oakland. The *Study Guide* is meant only as a review of materials students have already covered in coursework at school. The textbook used in Freddy’s ninth grade mathematics class [Fendel et al. 2003] covered probability over 30 days of games and experimentation, and was as clear as a textbook could be on the subject. Nevertheless, he could not remember anything of the concepts he learned in that course, nor could he relate what he found in the *Study Guide* to that experience. What was his problem? Was he simply intellectually incompetent, or was there something wrong with the textbook? Is he a hopeless case? Should he be graduating from high school if he can't pass a test like the CAHSEE, which is not really all that difficult?

I will argue that the problem lies neither in Freddy nor in the textbook. Freddy is no different from the many other students in the state who know English, but not the kind of English used in academic discourse, and especially in instructional texts. And there are problems with this textbook, to be sure, but not more so than with other texts addressed to students of Freddy's age. The passage is not so different from other instances of expository texts meant to convey information about subject matter such as probability and statistics for high school students. The problem is that students like Freddy — in fact, any student who has not learned the academic register used in written texts and in instructional discourse — need help learning it or they will find language to be an insuperable barrier to learning, whatever the subject matter. In mathematics education, it is a truism that the learner's language does not figure importantly, because "math has its own universal language; it is the same no matter what language is spoken by the students," as I am often reminded when I argue that mathematics teachers need to help their students who are English learners deal with the language in which math instruction is presented.

But is the language of mathematics universal? If students are unable to understand the words or the structures used in texts like the *Study Guide* we have been considering, could they understand the explanations offered in them? Would they be able to understand a teacher's explication of these ideas? Would they be able to solve the problems they encountered in high-stakes tests like the CAHSEE, which are often presented as word problems — simpler than the examples we have looked at from the CAHSEE *Study Guide*, but equally challenging linguistically? Consider, for example, the following released item from the 2004 CAHSEE [California 2004b, p. 25]:

Highway

		A	**B**	**C**
	1	**A1**	**B1**	**C1**
Road	**2**	**A2**	**B2**	**C2**
	3	**A3**	**B3**	**C3**
	4	**A4**	**B4**	**C4**

To get home from work, Curtis must get on one of the three highways that leave the city. He then has a choice of four different roads that lead to his house. In the diagram [to the left], each letter represents a highway, and each number represents a road.

If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?

A. $1/16$ B. $1/12$ C. $1/4$ D. $1/3$

A reader who understands the language and recognizes the format in which mathematics problems are presented might not have difficulty with this one, although a student trained to visualize the situation described in a word problem may experience time-wasting cognitive strain imagining a choice of three highways, each of which offers exits onto any of the same four roads. A student like Freddy, however, had no idea what was being asked for because he understood neither the concept of probability nor the reasoning called for in asking about the likelihood of an event, given the conditions presented in the problem. He had no clue as to how he might respond. And so he took a guess, having no idea as to why it might or might not be the correct answer. Would understanding the problem have helped? I will argue that no understanding of the problem was possible without an understanding of the concepts that figured in this problem, and there was no way Freddy could have understood those concepts without the language in which they can be explicated and discussed whether by a teacher in class or in the textbooks in class. Clearly, students like Freddy, a second language speaker of English, need greater familiarity with the language of textbooks and instructional discourse in order to deal with school subjects like mathematics. But in what ways is such language different from everyday English?

Academic English

There are various characterizations of academic English. The one I operate with is this: It is extended, reasoned discourse — it is more precise than ordinary spoken language in reference, and uses grammatical devices that allow speakers and writers to pack as much information as necessary for interpretation into coherent and logical sequences, resulting in greater structural complexity than one finds in ordinary conversational language. The differences stem from the functions such language has and the contexts in which it is used. It is often used in situations where there is an audience or a reader that may or may not have the prior knowledge or the background necessary to understand what is being talked or written about, and where the context itself offers little support for the interpretation of the text or talk. Even when using illustrations, charts, or diagrams, speakers or writers must rely on language to convey what they have to say to their audience or readers.

In the case of speakers, as opposed to writers, for example teachers explaining the concept of probability to high school students, there is the possibility of assessing the effectiveness of what they are saying by monitoring the students' faces while speaking. That kind of feedback enables speakers to rephrase, redirect, or otherwise tailor the discourse to facilitate successful communication.

The writer, however, does not have access to that kind of immediate feedback. In preparing an academic text of any sort (this chapter, for example), the writer has to build into the text itself, as much information as needed to communicate his or her message to anyone who is a likely reader of the text, given its purpose and intended readership. The writer can make some assumptions about what the intended readers are likely to know; such presumed shared background relieves him or her of having to build in background information, or to go into lengthy explanations. Anything that cannot be assumed must be explicated and contextualized.

In the case of this paper — one discussing language issues in mathematics education — the readers who are mathematics educators or mathematicians will not need to have any of the math concepts explicated (for example, no explanation is needed for "probability," a concept with which the readers are without question more familiar than the writer is). On the other hand, the grammatical and lexical features that get in the way of students understanding the language used in math texts or instruction may or may not be as familiar to such readers; hence, greater care needs to be taken in building into the discussion, explanations the writer hopes will allow the reader to follow what she is trying to communicate.

This is what gives academic discourse, whether written or spoken, its special character. Some other notable features of academic English can be seen in the two passages from the CAHSEE Study Guide [California 2004a] which were discussed above. Consider the sentence:

> *In probability, an event is a particular happening that may or may not occur.*

The need for specificity in academic discourse results in definitional clarity — in fields of study such as mathematics, ordinary words such as "event" sometimes have special meaning, and in such cases, they are stipulated as we see above: "an event is *a particular happening that may or may not occur*." Notice that the definition is more than a lexical equivalent: it is a noun phrase[4] stipulating the conditions that must be satisfied for "a particular happening" to qualify as "an event" in this usage. Academic language makes heavy use of complex noun phrases as we see in this definition. This noun phrase contains a relative clause ("*that may or may not occur*"), which is itself a sentence modifying '*happening*.'

Another feature of academic language is exemplified in the following excerpt from the CAHSEE *Study Guide* passage explaining what a dependent event is [California 2004b, Appendix, Math Vocabulary, p. 81]:

[4]A phrasal unit headed by a noun and which functions as a grammatical unit: some examples of noun phrases, from simple to complex: "Language," "the study of language," "the language on which you will be tested," "languages that are related to Latin."

> (But) some events are *dependent*; that is, the probability of one event depends on whether the other event occurs. For example, suppose you are randomly choosing two marbles, one after another, from a bag that contains three blue marbles and three red marbles. On your first draw, you have a 50% chance of drawing a blue marble. But on your second draw, the probability of drawing a blue marble depends on which color you pulled out on the first draw. The probability of getting a blue marble on your second draw is *dependent* upon the result of the first draw.

Here we see the use of the hypothetical introduced by "suppose": readers familiar with this construction know that they should be going into the hypothetical mode: what follows is a description of a set of actions that add up to a situation against which the rest of the text is to be understood. The actions described are complicated by the qualifier, 'randomly,' a word we recognize as a technical term in this context, but is not explained here. Reader are assumed to understand the conditions that the use of this word places on the actions being described: "Suppose you are randomly choosing two marbles, one after another, from a bag that contains three blue marbles and three red ones."

The question here is what do students like Freddy find difficult about such language? His ability to make sense of the definition contained in this passage depends on his ability to, in some sense, visualize the situation described given the description, understand the ways in which it is constrained, and then extrapolate the distinction between events that are said to be dependent from ones that are independent. On that hinges his ability to make sense of test items like the CAHSEE released problem discussed earlier: "If Curtis randomly chooses a route to travel home, what is the probability that he will travel Highway B and Road 4?" To do the reasoning called for, he has to understand each part of the problem as it is described, and to know what it means to choose a particular combination from multiple possibilities.

It is clear that Freddy needs a lot of help not only with mathematics, but with the language in which it is communicated as well. Without a massive effort from his tutors and from himself, he will do no better on the high school exit examination than he has in the past. He is discouraged, but he keeps working because he wants to learn and to complete high school.

What English Learners Need

Working with Freddy, one discovers what happens when language differences are not considered in learning. How did Freddy end up so limited in his command of English? How did he become so unprepared to handle even the limited mathematics tested in the high school exit examination?[5] Freddy is not unique. There are hundreds of thousands of other former English learners in high schools across the country with problems not unlike Freddy's. California happens to have the largest concentration of such students in its schools, where one in four students begins school with little or no English. California's solution to the problem has been to require that they be instructed entirely in English, by voter mandate.[6] But what happens when children try to learn complex subject matter — and mathematics is one such subject — in language they do not fully understand because they are in the process of learning it? Is it possible to learn mathematics without understanding the language in which it is being taught?

It may not be difficult at least during the early years of school for children to participate in instructional activities even though they do not fully understand the language in which instruction is given. It becomes problematic, once children reach the third or fourth grade where the concepts they are taught tend to be more complex and abstract, requiring more explanation, and therefore a greater understanding of the language used for explanation. If they are learning the language of instruction rapidly enough and are given ample support for dealing with the subject matter, their progress in mastering the subject matter will not be greatly hindered by their incomplete knowledge of the language.

But what of the many students who do not find it so easy to learn the language of instruction? In second language learning, much depends on access, opportunity, and support. What children learn depends on who is providing access to the language and support for learning it. If the source is mostly from other students who are also language learners, the version they end up with is a learner's incomplete version. If the source is mostly from teachers who are using the language for instructional purposes, and the students are encouraged to participate in the instructional discourse using that language, it is possible for them to acquire the academic register.

Freddy's lack of familiarity with academic English suggests that he did not have many such opportunities. In fact, tracing back over his school records, one sees that his problems with mathematics began early on, back in the early grades of elementary school, when he did not understand what it was he was supposed to have been learning. We can't know how he would have fared educationally had he been educated in a language he already knew and understood rather than in one he was trying to learn as a second language. It seems evident that he would have found it easier to learn what he was supposed to be learning in school,

[5] There is little in the mathematics section of the CAHSEE that is not covered in ninth grade mathematics.

[6] California voters passed Proposition 227 in 1998, greatly restricting the use of bilingual instruction in schools except under special circumstances. Voters in Arizona followed suit in 2000, with the passage of Proposition 203, and Massachusetts voters passed legislation in 2002 that altogether bans bilingual education in that state.

had he been instructed in Spanish. There is some intriguing evidence from comparisons of the academic performance of immigrant students who received some formal education before coming to the United States and that of non-immigrant students of the same ethnic, socio-economic status, and linguistic backgrounds, with the immigrants outperforming the non-immigrant students [Berg and Kain 2003; Thomas and Collier 1997]. Cummins [1981] found that the older the immigrant students were at the time they arrived in Canada the better they performed in Canadian schools, both in measures of second language learning and in academic learning.

Ideally, Freddy (and students like him) would learn subject matter like mathematics in his primary language in elementary school, while receiving the instructional support required for mastering English. With a firm grounding in the foundational concepts and operations in math, he could easily enough transfer what he knows to mathematics classes taught in English when he was ready for that. Much depends on whether students like Freddy receive instructional support to develop the academic register for mathematics in school. In an ongoing experimental program, Boston University researchers, Suzanne Chapin and Catherine O'Connor [2004] have been demonstrating what a difference such support can make both in the mathematics and English language development of English learners in Chelsea, Massachusetts middle schools.

Working with mathematics teachers in the Chelsea schools, the researchers designed a mathematics program to promote the language and mathematical skills that are typical of higher performing students. The mathematics program met both the state of Massachusetts's mathematics standards and the standards adopted by the National Council of Teachers of Mathematics. Project Challenge, as the program was called, recruited at-risk minority, economically disadvantaged middle school students who had potential and an interest in mathematics. Most of these children, while average on all indicators used by the project for selecting students, had significant gaps in their mathematical and linguistic knowledge. For example, many of these inner-city students had difficulty dealing with fractions, and they also had difficulty applying the mathematics they did know to activities outside of school, such as in sports. Two-thirds of the students recruited for the program were English learners, and entered the program with limited English proficiency. Most of the English learners were Latinos, and the rest were members of other under-achieving minority groups.

During the first four years of the project, the researchers and teachers worked with nearly 400 students, from fourth through seventh grade. During those years, the children were presented with a challenging math curriculum designed to develop their understanding of mathematics, and to improve their problem-solving abilities in dealing with complex mathematics problems. They received an hour each day of instructional support from their teachers to verbalize their evolving understanding of the concepts and operations they were learning. The emphasis was on getting the children to take responsibility for understanding and explaining their reasoning in solving mathematics problems, all in the language they were just learning. The focus of this instruction was on explanation, generalization, and justification of mathematical concepts and procedures. The teachers taught the children to explain, defend, and discuss their mathematical thinking with their classmates. By 2002, the team had outstanding results to report: using measures such as the mathematics subtests of the California Achievement Test, and the Massachusetts Comprehensive Assessment System, Project Challenge students were posting mean scores in the 87th percentile, outperforming children in the state of Massachusetts as a whole, and even those in higher performing districts in the state.

This is the kind of instructional support English learners like Freddy have needed. What it takes is attention to language in mathematics instruction, and indeed across the curriculum. It begins with educators recognizing that the difficulty that English learners have in subjects like mathematics goes beyond the subject matter itself.

How meaningful is any assessment of Freddy's mathematical competence, whether in the classroom or on a formal test such as the CAHSEE, given his present linguistic limitations? As Alan Schoenfeld points out (personal communication, August 16, 2005): "you can't know what he knows mathematically unless he is given an opportunity to show it, which means being given problems whose statements he can understand." Freddy may be capable of doing the mathematics he is expected to handle, if he is able to get past the linguistic demands of the problems. Hence attending to such demands must be an essential component of assessment and of instruction.

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