Section 6
The Importance of Societal Context

To invert Vince Lombardi: context isn't the only thing, it's everything. As the six chapters in this section show, it is sometimes the case that the best way to look inward is to begin by looking outward. Just as fish are said to be unaware of the water in which they swim, those embedded in a cultural context may be unaware of just how much of what they do is socially and culturally determined, and not simply "the way it is" or "the way it must be."

In Chapter 17, Michèle Artigue takes readers on a tour of mathematics assessments in France. As she does, it becomes clear that some aspects of assessment are universal and that some are very particular to local circumstances. The broad goals of mathematics assessment discussed by Artigue are parallel to those discussed elsewhere in this volume. Assessments should, she writes, reflect our deeply held mathematical values; they should help teachers to know what their students are understanding, in order to enhance diagnosis and remediation; they should provide the government with a general picture of the country based on representative samples of pupils. Note that different assessments in the U.S. serve these functions. For example, the National Assessment of Educational Progress (NAEP) provides some "benchmarking" of American student performance. However, NAEP provides no direct feedback to teachers or students. Numerous high-stakes tests at the state level — in some cases, at every grade from second grade on — are highly consequential for students but have no diagnostic value. (No other nation tests students as much and with as little useful feedback as the U.S. It is the case that local assessments such as those discussed in Chapters 10, 12, and 14 do provide diagnostic information at the student, classroom, and district levels. However, they are not used on a larger scale.)

Artigue also discusses the fact that, in France, assessments given before the end of high school are explicitly conceptualized as a mechanism for systemic change and for teacher professional development. The impact of such "levers for change" in a coherently organized system — in France the Ministry of Education essentially determines curricula nationwide — is very different than the impact in a distributed assessment system such as in the United States, where each of the fifty states has its own assessment standards and the nation's 15,000 relatively independent school districts can set about meeting their state's standards with a fair degree of latitude.

In Chapter 18, Mark Wilson and Claus Carstensen take us to the shores of science learning and of assessment design more generally; then they bring us back to mathematics. Originally the Berkeley Evaluation and Assessment Research Assessment system was designed to provide diagnostic and conceptual information about students' understanding of science concepts. The system is grounded in the "assessment triangle" put forth by the National Research Council (2001) in *Knowing What Students Know*, and in four building blocks for constructing an assessment system: (1) examining the growth of knowledge over time (a "developmental perspective"), (2) seeking a close match between instruction and assessment, (3) generating high-quality evidence, that (4) yields information that is useful to teachers and students, and can feed back into instruction. (In Chapter 10, we have seen examples of how (3) and (4) might happen.) Wilson and Carstensen describe the recent application of the system to mathematics assessments.

Chapters 19 and 20 explore issues of mathematics assessment, teaching, and learning for students whose first language is not English. To make the extreme case related to Lily Wong Fillmore's argument in Chapter 19: how well would you do on a mathematics assessment administered in Swahili, and what would your responses reveal about your understandings of mathematics? Also to the point, how long would it take you to get up to speed if you were being taught mathematics in Swahili? To varying degrees, this is the dilemma being faced by the myriad students in U.S. schools for whom English is not yet a comfortable language. In Chapter 20, Judit Moschkovich addresses some related themes. If a student's mathematical understandings are assessed in a foreign tongue, that student's competencies may not be revealed. That is, the person being assessed may well know some things, but he or she may be unable to express them in the language used for the assessment. Note that in this case, providing remediation on content related to the assessment is precisely the wrong thing to do! In addition, Moschkovich makes a convincing case that diagnosis and remediation on the basis of vocabulary is far too narrow. She discusses the linguistic resources that students bring to their mathematical encounters, and the ways that we can recognize and capitalize on them.

The final two chapters in this volume, by Elizabeth Taleporos and Elizabeth Stage, focus on external, systemic realities — on the ways that political contexts shape assessment and its impact. In Chapter 21, Taleporos describes some of the history of assessment and its impact in New York City over the past few decades. Unsurprisingly, people who get mixed messages about what is expected of them get confused — and the confusion breeds problems. Taleporos describes such conflicts in New York in the 1980s, where statewide assessments focused on minimal competency while city-wide assessments focused on the percentage of students who performed above average on a norm-referenced nation-wide test. Chaos resulted. Later on, when testing in the system got more consistent, the situation improved. (There are lessons to be learned here, given the plethora of testing requirements catalyzed by the No Child Left Behind Act.) In Chapter 22, Stage reviews some of the history of California's statewide mathematics assessments. Stage describes the origins of the California Assessment Program, a low-stakes testing regime that provided teachers and school districts with useful information about what students were learning in their classrooms and schools. Political pressures raised the stakes, and with higher stakes came greater statistical constraints and more narrowly defined items. These are very consequential changes: as the title of Stage's chapter suggests, the items that teachers see as exemplifying the test have a significant impact in shaping what they teach. This is not always a good thing.

When it comes to assessment, then, we are living the ancient curse: these are indeed interesting times. There is much to learn if we heed the lessons of the chapters in this section.

Assessing Mathematical Proficiency
MSRI Publications
Volume **53**, 2007

Chapter 17
Assessment in France

MICHÈLE ARTIGUE

Introduction

This chapter provides an overview of the methods currently used in France to assess students' mathematics learning, of the changes occurring in this area, and the rationale for them. As in any country, the methods of assessment used in France are part of the country's general educational culture. In order to understand them, it is helpful to know about the characteristics of this culture. I briefly describe these cultural characteristics, before discussing the methods of assessment. These include both internal assessments done by teachers in classrooms and external assessments.[1] In this chapter, I will focus on the external assessments, in particular, on two different kinds of assessment: the *Baccalauréat*, which is the national examination at the end of high school, and the national diagnostic assessment at the beginning of middle school.[2] Then, after briefly discussing current research on alternative modes of assessment relying on technology, I end this chapter with some general comments on assessment issues.

Some Characteristics of the French Educational System

Students in France begin secondary education at age eleven, after two to three years of kindergarten[3] and five years of elementary school. Secondary education

[1] As in any country, the situation is in flux; by the time this volume appears, some things will have changed. Readers who speak French can find current information at the Web sites listed below.

[2] Updated statistics from the *Baccalauréat* can be found on the Web site of the French Ministry of Education: http://www.education.gouv.fr/stateval/. The diagnostic assessments discussed in this paper are accessible on the Web site http://cisad.adc.education.fr/eval/.

[3] In France, the term equivalent to kindergarten, *maternelle*, also designates preschool. Available to children aged from two to five, it is not compulsory — yet almost 100% of children aged three to five attend it. It is regulated by the French Department of Education.

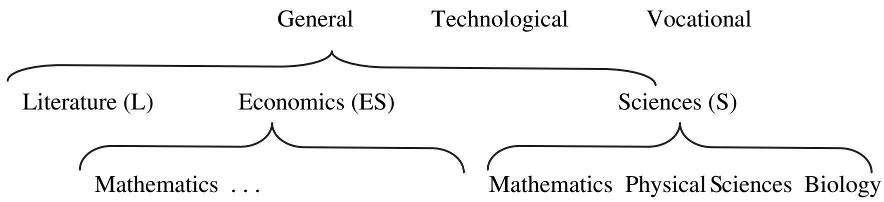


Figure 1. Differentiation in high school. Top row: grade 10 (streams). Middle row: grade 11 (orientations). Bottom row: grade 12 (specialties).

is comprised of two parts: first, a four-year program called *collège* (which I will call "middle school"), and then the *lycée* (which I will call "high school") beginning at grade 10. Compulsory education ends at age sixteen, and thus includes the first year of high school. Prior to high school, the curriculum is the same for all students.[4] In high school, there are three main streams: the general and the technological streams which are three-year programs, and the vocational which includes two successive two-year programs. At present, approximately half of the high school population is in a technological or vocational stream. Each stream is further differentiated. For the general stream for instance, differentiation occurs at grade 11. Then students can choose between three different orientations: Literature (L), Economic Sciences (ES) and Sciences (S); at grade 12 (the last year of high school), they have also to choose a speciality. In S for instance, this can be mathematics, physical sciences, or biology (see Figure 1). The different high school programs all end with a national examination: the *Baccalauréat* (mentioned above) which allows students to enter post-secondary education. However, some post-secondary programs such as the CPGE (specific classes preparing students for national competitions for the most prestigious higher institutions such as the *Ecole Polytechnique* and the *Ecoles Normales Supérieures*, as well as for many engineering and business schools) select their students on the basis of their academic results in high school, before knowing their results for the *Baccalauréat*.

France has a national curriculum. Until 2005, the 21-member national council for curriculum (CNP) designed the general curricular organization and prepared specific guidelines for each discipline; group of experts created the syllabuses for each discipline. These syllabuses were then submitted for approval to the CNP and to various other authorities. As of 2005, the CNP was replaced by the 9-member Haut Conseil de l'Education (HCE), which has less content-specific expertise.

The mathematics curriculum is integrated in the sense that chapters on algebra and geometry alternate in textbooks. Synthetic geometry still occupies a rather

[4]Specific instititutions exist for students with disabilities.

important place and, in it, transformations play an essential role. Due to history, and to the introduction of the metric system during the French Revolution, in the curriculum for elementary school and even in the first years of middle school, numbers with finite decimal representations play a more important role than rational numbers in general, and fraction representations are not really used in elementary school. Mathematics occupies about 5.5 hours per week in elementary school and about 3.5 in middle school. Time allotment varies in high school according to the different streams and orientations. Table 1 shows the respective timetables for the general stream in high school. Note that mathematics is optional for the last year in L (this is the only class where it is optional in secondary schools).

Grade and Orientation	Timetable						Workload (range)
	C	M	AI	TD	O	S	
10	3 hr	1 hr	1 hr				4–5 hrs
11 L	1 hr			1 hr	3 hr		2–5 hrs
11 ES	2.5 hr			0.5 hr	2 hr		3–5 hrs
11 S	4 hr			1 hr			5 hrs
12 L					3 hr		0–3 hrs
12 ES	4 hr					2 hr	4–6 hrs
12 S	4.5 hr			1 hr		2 hr	5.5–7.5 hrs

Table 1. Current mathematics allotment for the general stream. C means "Cours"; M, "Modules"; TD, "Travaux dirigés"; O, "Option"; S, "Specialité" and AI, "Aide individualisée." "Cours" denotes whole-class activities, "Modules" and "Travaux dirigés," half-class activities, "Option," optional courses, "Specialité," the compulsory speciality courses, "Aide individualisée," remedial activities organized for groups of at most eight students.

In high school, the syllabus also varies according to the different streams intended to serve the diversity of students' interests and needs.[5] For instance, the L syllabus for grade 11, called *Informatique et Mathématiques,* focuses on mathematics used in a visible way in society: tables of numbers, percentages, some statistical parameters, and graphical representations; it encourages the systematic use of spreadsheets. The ES syllabus includes more statistics than the others and an option for the speciality in grade 12 includes graph theory, which

[5]The official descriptions are accessible on the Web site of the Ministry of Education (www.education.gouv.fr) in French. For overviews in English, the reader can consult [Artigue 2003] or the description of the French system prepared by the CFEM, which is the French subcommission of the International Commission on Mathematical Instruction, for ICME-10, which is accessible on the Web site of the CFEM, http://www.cfem.asso.fr/syseden.html.

does not appear in the other syllabuses. In the S syllabus calculus plays a larger role and the mathematics speciality includes number theory, a topic that does not occur in others.

As mentioned above, mathematics learning in France is assessed in different ways, both inside and outside the classroom. High school teachers are in charge of formative and summative assessments for their courses. The results of these assessments serve to decide if the students will have to repeat courses or not, and to decide which high school programs they will take. Beyond that, there are external modes of assessment. These include some traditional examinations such as the *Brevet des Collèges* at the end of middle school, the *Certificat d'Aptitude Professionnelle* and the *Brevet d'Enseignement Professionnel* at the end of the first two-year program in vocational high schools, and of course the *Baccalauréat*. About fifteen years ago, diagnostic tests taken at the beginning of the academic year were introduced for some students. In this chapter, I will focus on the external assessments, beginning with the traditional examinations.

Traditional Examinations: The Case of the *Baccalauréat*

The first examination students take is the *Brevet des Collèges*. This is an examination taken at the end of middle school that combines written tests in French, Mathematics, Geography, and History, and the results of the internal evaluation of the students in grades 8 and 9. This examination is not compulsory and results do not affect students' choice of high school programs. In 2003, it was taken by about 700,000 students and the overall success rate was 78.3% (82% for girls and 75% for boys).

The most important examination is the *Baccalauréat*. This is a national examination at the end of high school, and it is necessary to pass it to enter post-secondary education. About 69% of the population now attend the last year of high school (75.4% for girls and 63% for boys) and take the *Baccalauréat*. In 2002, the 628,875 candidates for the *Baccalauréat* were distributed as follows: general *Baccalauréat* 52.2%, technological *Baccalauréat* 29.3%, vocational *Baccalauréat* 18.5%.

Overall pass rates for the different streams in 2003 are given in Table 2.

In the same year, in the general stream, pass rates for the different orientations were: 82.2% in L, 79.4% in ES, and 80.1% in S. Girls represented 45.6% of the new *bacheliers* in S, 65.7% in ES, and 83.6% in L. Pass rates increased sharply over the past twenty years (in 1980, only 34% of an age-cohort attended the last year of high school) but they have now stabilized.

The duration and the content of the mathematics part of the examination is different in the different streams and orientations, as is the weight of mathematics in the global mean which decides of the attribution of the *Baccalauréat*.

Stream	Girls	Boys	Total
General	82.3%	77.5%	80.3%
Technological	79.8%	73.8%	76.8%
Vocational	78.6%	75.2%	76.6%
Total	81.0%	75.8%	78.6%

Table 2. 2003 pass rate for the *Baccalauréat*.

Table 3 gives the corresponding statistics for the general stream. As can be seen, even in the S orientation, the weight of mathematics is only about 20%.

Traditionally, the mathematics test consists of two exercises and one problem with several parts, the whole covering important parts of the syllabus. Exercises may involve different topics: number theory (in S with mathematics speciality), complex numbers, two- and three-dimensional geometry, probability and statistics, sequences, graph theory (in ES with mathematics speciality). One of the exercises is different for those having choosen the mathematics speciality in ES or S. The problem itself generally deals with functions and calculus, and in ES it is often motivated by an economics context.

In order to give the reader a precise idea of what such a test might be, I have translated the 2003 test given to the students with orientation S who specialized in mathematics[6] (see Appendix 1). This test follows the traditional structure but was considered very difficult by students and teachers. On the one hand, it was the first test aligned with the new high school curriculum,[7] and on the other hand while globally obeying the traditional test structure, it did not respect some traditional standards. Thus, it is a good illustration of what is standard in such a test and what is not. The examination was the source of passionate

	Written test duration in hours	Coefficient
L	1.5 (taken at grade 11)	2 out of 38
L	3 (if option taken in grade 12)	4 out of 34
ES	3	5 out of 37 (+2 if speciality)
S	4	7 out of 38 (+2 if speciality)

Table 3. Weight of mathematics in the G stream

[6]The test, referred to as the *metropolitan* test, is given in continental France. The tests given in the overseas territories and in foreign centers did not generate the same discussions.

[7]The new high school curriculum was introduced in grade 10 in 2000–2001, and thus reached grade 12 in 2002–2003.

discussions all over the country and in the media. Indeed, the Minister of Education himself had to make a statement on national TV. The importance given to the *Baccalauréat* in French culture, the fact that the scientific orientation is still considered the elite orientation certainly contributed to the strength of the reactions.

Analyzing the metropolitan test for the S orientation in 2003. The first exercise has a traditional form. Complex numbers are used for proving geometrical properties: showing that a given triangle is a right isosceles triangle and that given points lie on the same line. A geometrical transformation is involved. In this case, it is a rotation (it could have been a similarity) and students have to express it as a complex transformation. Students have to use different representations for complex numbers (cartesian and exponential) and to express the image of a set of complex numbers (a circle, in this case) under the transformation. Such an exercise cannot be solved without a reasonable knowledge of complex numbers and connections between complex numbers and geometry, but these are standard expectations. Moreover, students are carefully guided by a lot of subquestions, can check their answers by reasoning on a geometrical figure, and the computations are not technically very complex. This is also standard.

Exercise 2 is specific to the mathematics speciality[8] and connects three-dimensional geometry and number theory. The latter is not so traditional because three-dimensional geometry is not frequently examined in the *Baccalauréat*. Many teachers do not feel comfortable with it or do not like it, and when they lack time, they often skip this part of the course in high school. Students are thus less prepared to solve problems in three-dimensional geometry. Moreover the test questions cannot be considered routine. This is especially the case for Question 2 whose form is not common and whose difficulty could have been easily limited by saying that the planes are parallel to coordinate planes instead of parallel to a coordinate axis. The last part including number theory is not completely new in spirit because number theory has sometimes been linked to a geometrical context in such *Baccalauréat* exercises, but generally it has been related to two- rather than three-dimensional geometry. What is new is the expected reasoning based on computations using congruences (which have just entered the curriculum), the autonomy given to the students in selecting a solution method for solving Question 3 (which is nonroutine), and the fact that solving of Question 4 involves an infinite descent process or reasoning by

[8]The second exercise for the students who had not chosen the mathematics speciality dealt also with three-dimensional geometry. Students were asked to study different properties of a tetrahedron OABCD whose faces OAB, OAC, and OBC were right triangles (with right angle at O), and verifying that OA = OB = OC. This exercise, which used knowledge of three-dimensional geometry taught two years before in grade 10, was more criticized than the speciality exercise.

contradiction and minimality. These are not familiar forms of reasoning for the students and the question format does not guide them.

What was in fact most criticized on this test was not Exercise 2, even if it was considered rather difficult, but the problem. At first look, this problem could be considered standard — it deals with calculus, has a long multipart text, and students are guided by a lot of questions and hints. Nevertheless, it is far from standard. First of all, it reflects some new aspects of the curriculum, especially the emphasis on relationships between different scientific disciplines and on modeling activities. The problem tests students' knowledge of calculus as usual, but by asking them to use this knowledge in the analysis of a situation in biology. Two competing models are given for representing the increase in a population of bacteria, and students after studying these two models are asked to judge their adequacy to represent some empirical data. This by itself would have been enough to make this problem unfamiliar because students would not find anything similar in the *Annales du Baccalauréat* traditionally used to prepare for the examination. This unfamiliarity, nevertheless, was not the only cause of the violent reaction the problem generated. What was strongly criticized was the way the problem was set up and made more complicated by the introduction from the beginning of many parameters (in the French secondary curriculum, the use of parameters is very limited and students are not used to carrying out symbolic computations with many parameters). Moreover, the problem did not follow the usual tradition that the beginning of each part is straightforward and that the difficulty increases progressively. For instance, Question 1a in Part B asked the students to make a change of unknown in a differential equation. This is a complex task which requires having an elaborated conceptualization of the idea of function. For many students this was undoubtedly the first time they encountered such a task (the study of differential equations is only a small part of the syllabus, and students only have to know how to solve very simple linear differential equations with constant coefficients). Question 2d was also criticized, this time because it did not make sense in this situation, and seemed to have been introduced only to check that students could relate mean and integral, and were able to calculate the antiderivative of the function g.

Teachers and students are used to the fact that items in the *Baccalauréat* are long, occupying several pages, and that they cover a wide part of the syllabus. They are also used to items where the autonomy of the student is rather limited. The problems posed are not generally easy to solve but many subquestions and intermediate answers help the students, enabling them to continue even if they have not succeeded at solving a particular question. The 2003 test items for the orientation S, while it did have many of these characteristics, was much more cognitively demanding. This, together with the existence of the new syllabus for the test, deeply destabilized teachers and students. This example raises unavoidable questions about what is really assessed or can be really assessed through this kind of examination, and about the distinction between what is routine and what is nonroutine — a distinction which is necessarily fuzzy and dependent on institutions and educational cultures.

Current debates and changes. The *Baccalauréat* appears today to be a very burdensome and costly system of evaluation with both positive and negative effects. Its national and external character is generally seen as positive, together with the fact that the desire to pass the exam motivates students to work. But its existence makes the academic year one month shorter because high school teachers have to administer the tests, correct the tests, and organize oral examinations for those whose score lies between 8 and 10. Because of the test's national character, a minor local incident may have profound effects. Moreover, mathematics teaching in grade 12 is, in many places, too strongly oriented toward preparation for the test, and the types of knowledge that this kind of test cannot assess tend to be neglected. Nevertheless, changing the *Baccalauréat* appears very difficult. The *Baccalauréat* is like a monument, a myth, which is part of the culture. Each time the possibility of change is mentioned, impassioned reactions come from all over. Commissions have been created, different possibilities have been discussed (and some locally tested): replacing the present structure by four or five exercises in order to avoid the constraints resulting from a long problem, giving more autonomy to the students with shorter texts and favoring more reflective work, combining written tests with other forms of assessment …but, no substantial changes have occurred.

The events of 2003 did have some consequences. The Ministry of Education decided to prepare secondary education for some unavoidable changes. The General Inspectorate, an institutional body of general inspectors that is responsible for the *Baccalauréat*, was asked to prepare exercises aligned with the new curriculum, which were more diverse in both form and content, and better aligned with the role played by technology in mathematics education today. A first set was put on-line in December 2003[9] and has been discussed in regional meetings with inspectors and in teacher training sessions. Translations of two of its exercises appear in Appendix 2.

As of 2004 the *Baccalauréat* assessments have been effectively structured around four to five exercises. In addition, in 2005, new items in which students are asked to prove results that are from the syllabus were introduced. These items, known as ROC (Restitution Organisée de Connaissance) items, have been

[9] See www.education.gouv.fr

controversial. Examples can be downloaded from the Ministry Web site, http://eduscol.education.fr/D0056/accessujets.htm.

(In 2005, the Minister of Education tried to introduce some substantial changes in the *Baccalauréat* (for example, assessing some disciplines through the results of internal assessment as is the case for the *Brevet des Collèges*) but he had to give up these plans because of strong student opposition. Students argued that the *Baccalauréat* would lose its national character and that the diplomas would be given different value depending on which high schools students came from.)

Further, a change in assessment methods resulted from the introduction of the TPE (supervised project work). This new didactic organization was introduced in grade 11 in 2000–2001 and extended to grade 12 the following year. Working in small groups over one semester, students work on a collective product, starting from varied resources, on a subject chosen by them from a list of national topics. The TPE must involve at least two disciplines, including one required for the students' high school program. The students' project is supervised by teachers from the disciplines concerned. Two hours per week are reserved for the TPE in the students' timetable.

This new didactic organization has several aims:

- providing students with the opportunity to develop a multidisciplinary approach to questions which are not just school questions;
- helping them to mobilize their academic knowledge in such a context;
- broadening their intellectual curiosity;
- developing their autonomy;
- helping them to acquire methods and the competencies required for working in groups;
- developing the abilities necessary for an effective search, selection, and critical analysis of documentary resources;
- and, finally, establishing more open relationships between teachers and students.

This work and the scientific competencies it allows students to build cannot be assessed by the standard examination. In fact, its evaluation takes into account the production of the group of students as well as their written and oral presentations. As of 2005, the TPE only exists in grade 11. Students can choose to have their TPE evaluation taken into account as part of their *Baccalauréat* evaluation. As this chapter goes to press, new forms of assessment for the *Baccalauréat*, which are more experimental and require intensive use of technology, are being explored.

As explained above, even if among the different methods of assessment, external examinations are considered most important, there are other ways to assess students' mathematical knowledge. In the next section quite different assessment methods are discussed: the diagnostic tests which were introduced about fifteen years ago.

Diagnostic Assessment: The Case of the Grade 6 Test

Diagnostic tests at the beginning of the academic year were introduced at the start of middle school and high school about fifteen years ago. For years, they were compulsory in grade 3 and grade 6; in 2002, they were extended to grade 7. They concern both French and Mathematics.

The diagnostic assessment, carried out by the DEP (Direction of Evaluation and the Future) at the Ministry of Education, has two main functions:

- providing the Ministry with a general picture of the country based on representative samples of pupils. The pupils' scores, analyzed according to different socio-demographic variables such as age, sex, nationality, parents' profession, are published every year and are also accessible on-line[10];
- providing teachers with a tool allowing them to better identify and answer the mathematical needs of the different categories of their pupils.

But the educational institutions would like it to be also:

- a way for accompanying curricular changes and influencing teachers' practices;
- a tool to help the regional academic institutions appreciate local needs for teacher training and to develop appropriate training programs.

In the following, I will focus on the diagnostic assessment administered when students enter secondary school in grade 6. The assessment items change every year but keep the same structure and form. The assessment is a collection of short exercises (in 2002, 39 exercises out of 77 questions) mixing multiple-choice and constructed-response items (some appear in Appendix 3). Thus they are more similar in format to U.S. tests than are the traditional French examinations. Items are designed according to a matrix of general competencies and mathematical domains. Table 4 gives the test structure and results for 2002.

Note that the standard deviation is rather high. The top ten percent of the students have a success rate of 91.9% while the bottom ten percent only reach 28%. There are also important differences in scores between middle schools when classified by social characteristics: the mean score of the middle schools located in ZEP (priority area for education) is only 52%. We also note that girls outperform boys in French, but not in Mathematics (66.3% versus 63%).

The test is administered during three ordinary 50-minute classroom sessions during the second and third weeks of the academic year. Middle schools receive

[10] See www.education.gouv.fr/Evace26

Domains	number of items	mean	median	standard deviation
Numeration and number notation	17	65.2	70.6	21.8
Operations	18	69.9	72.2	21.5
Numerical problems	6	65.2	66.7	29.6
Geometrical tasks	20	61.6	65	21.8
Data processing	16	63.3	68.7	23.3
Total score		65	67.5	19.6
Competencies				
Seek information, interpret, reformulate	8	61.7	62.5	24.9
Analyze a situation, organize a process	29	64.3	65.5	22
Produce an answer, justify it	10	57.7	60	24.9
Apply a technique	7	78	85.7	19.8
Directly use a piece of knowledge	23	66.1	69.6	22.1

Table 4. Mathematics test structure and 2002 scores.

from the Ministry of Education as many booklets with the different exercises as they have pupils, and specific booklets for the teachers. The latter explain the general structure of the test, test administration conditions (for instance, the amount of time given to pupils for solving specific exercises or groups of exercises in order to increase uniformity in testing conditions), the aim of each exercise and the competencies it is intended to test, how to score the answers, and what kind of help can be provided to the pupils who fail.

The teachers supervise the administration of the test, and then enter their students' answers into a document using specific software. The documents are then sent to the DEP which selects representative samples and produces an overall statistical analysis. By using the software, teachers can access the results of their pupils. They can also use it to obtain different statistics, and to identify groups of students according to characteristics of their answers. Middle schools are also asked to meet with the pupils' parents in order to present to them the results of the evaluation.

The test does not attempt to cover the whole elementary syllabus. Teachers are invited to use the on-line database for previous tests to find complementary items if they want to have a more accurate vision of the state of their class when beginning particular chapters. Moreover, having in view formative rather than summative assessment, the test designers include tasks associated with competencies that students are not expected to have fully mastered by the end

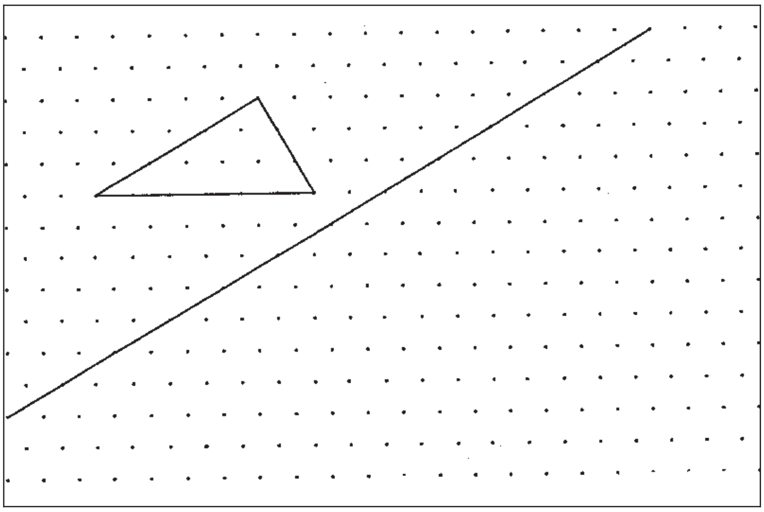


Figure 2. Illustration of task asking for reflection of a triangle.

of elementary school. This is, for instance, the case for the task illustrated in Figure 2 which asks for the construction of the reflection of a triangle.

In elementary school, students become familiar with tasks that involve drawing reflections of figures reflected across a horizontal or vertical axis on rectangular grids. For this, perception is very helpful. In contrast, the task on the diagnostic test uses a grid of points that form a triangular network and the axis of reflection is neither horizontal nor vertical (see Figure 2). Drawing the image requires more than perception, students have to use the mathematical properties of reflection in an analytical way. Didactic research shows that different kinds of errors can be expected in such a context, for instance, mixtures between reflection and translation. Some of these are given in the teachers' booklets in order to enable teachers become aware of them. Such tasks can thus contribute to showing the teachers the new competencies which must be gained in grade 6 with respect to reflection. The same can be said for tasks involving the invariance of area under decomposition and recomposition as in the exercise shown in Figure 3; and also for tasks asking for justifications about the nature of quadrilaterals or for tasks involving decimals (see Appendix 3).

There is no doubt that this diagnostic assessment offers a view of assessment which is very different from that described for the *Baccalauréat*. The tools that were developed for it in the last fifteen years, the collected data and their statistical analysis are clearly useful for understanding and adapting instruction to pupils' mathematical knowledge upon entering middle school. As mentioned above, information from diagnostic tests can also help smooth the transition between the cultures of elementary and secondary school, by pointing out some

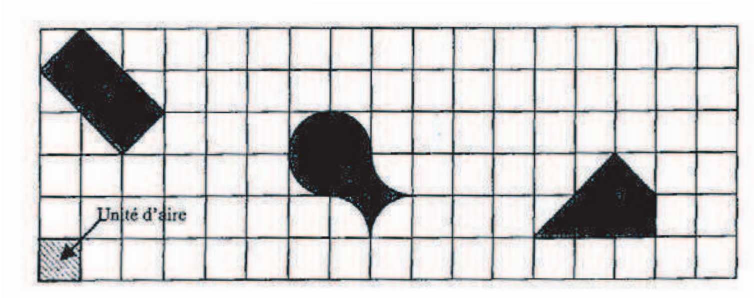


Figure 3. Expressing areas of figures with respect to a given unit of area.

necessary evolutions in the institutional relationship to mathematics, and by making middle school teachers sensitive to the fact that many mathematical notions introduced at elementary school are still under construction. At a more global level, the results of the diagnostic test display what is hidden by a score given by a single number: 65% which can be considered acceptable because the scope of the test is not limited to notions that must be fully mastered at the end of elementary school. The test results also display differences between performance in different areas and competencies (see Table 4), but, most importantly, differences between the top ten and the bottom ten percent of students, between middle schools in low-income areas and others. Even at the beginning of middle school, strong educational inequalities exist in France and must be addressed. The information I collected when preparing this chapter left me with the feeling that the diagnostic test is not used as well as it could be, and that we could benefit more from the investment made in its development. It is the case that teacher training sessions have been regularly organized with both elementary teachers and secondary teachers in order to use the results of the diagnostic test to promote reflection on the transition from elementary to secondary school; and, to help secondary teachers reconsider classroom management and remediation. Nonetheless, these sessions do not seem to have seriously influenced teachers' practices.

Understanding the precise effects of the different methods of assessment on the learning and teaching of mathematics, overcoming the limitations of most current assessment tools, and using the information they give us in order to improve mathematics teaching and learning is not easy. It surely requires research. In the next section, I describe briefly a research project that I am involved in, whose aim is to develop diagnostic and remedial tools in a specific domain — algebra — by relying on the new possibilities offered by technology.

Looking Towards the Future: The LINGOT Project

The LINGOT project is a multidisciplinary effort involving researchers in mathematics education, artificial intelligence, and cognitive ergonomy.[11] Its origin was a multidimensional model for considering competence in elementary algebra developed by Grugeon in her doctoral thesis [Grugeon 1995]. To this multidimensional model was attached a set of 20 tasks which allowed the exploration of the students' algebraic competence according to the various dimensions of the model. One ambition of this model and of the associated set of tasks is to allow the identification of lines of coherence in students' algebraic behavior, which are then used in order to design appropriate didactic strategies. The empirical part of the doctoral thesis showed how productive this model was in identifying such lines of coherence and helping students who had difficulty with algebra.

But the diagnosis was burdensome and rather sophisticated. Collaboration with researchers in artificial intelligence offered new perspectives. Building a computer version of the set of tasks, automatically coding responses to multiple-choice tasks, and designing ways to help teachers code other responses could certainly lighten the diagnostic process. This was the goal of the PEPITE project.

The LINGOT project currently under development is a third phase [Delozanne et al. 2003]. One aim is to transform the diagnostic tool provided by PEPITE into a flexible one which can be adapted by the teachers for their particular aims without losing the automated diagnosis. Another aim is to reduce the number of tasks necessary for diagnosis by relying on a dynamic diagnosis using Bayesian models. As requested by the teachers who piloted PEPITE, the LINGOT project intends to link the diagnosis to didactic strategies by associating to it potential learning trajectories. A final aim is to understand how teachers can appropriate such tools — which are at variance with their usual diagnostic methods — and transform these into professional instruments. Of course, in the project all these dimensions mutually intertwine. We are far from the end of this project, and the difficulties we encounter often seem barely surmountable, but at the same time, I have the feeling that it helps us to better understand the cognitive and institutional complexity of assessment issues.

Concluding Comments and Questions

In this chapter, I have briefly presented some of the methods used in France to assess students' mathematical knowledge, focusing on external examinations,

[11] For a description of the PEPITE and LINGOT projects see http://pepite.univ-lemans.fr/. There is a link to a program description for English speakers.

but balancing the contribution between two very different types of assessment: (1) the traditional examination whose paradigmatic example is the *Baccalauréat*, and (2) the diagnostic test whose paradigmatic example is the test taken by pupils on entering middle school. Of course, these two types of assessment do not have the same function. The *Baccalauréat* is a summative evaluation, and the grade 6 test is a diagnostic evaluation which is situated within a formative perspective. Due to its summative character and its institutional importance — it is a condition for entrance into post-secondary education and gives students the right to enter a university — the *Baccalauréat* strongly influences teaching in the last year of high school. The grade 6 test does not have a similar influence on teaching in grade 5, the last year of elementary school. These two tests raise common issues which are general issues relevant for any form of assessment. I will mainly focus on two of these general issues:

- How can one make assessment reflect fundamental characteristics of mathematical knowledge such as the following: the diversity of its facets on the one hand, the fact that it is to a large extent contextualized knowledge on the other?
- How can one make assessment reflect the values we want to transmit in mathematics education?

Students' mathematical knowledge is not easy to understand and is inherently multidimensional. Researchers have built different models in order to describe this multidimensionality; for example, the model created by Kilpatrick was mentioned several times during the MSRI conference [NRC 2001]. Some dimensions seem *a priori* easier to understand than others: for instance, investigating technical competencies seems easier than investigating mathematical attitude or creativity. But even technical competencies are not so easy to determine. Small variations in the labelling of the tasks, or in their didactic variables can generate large differences in students' responses. The boundary between what is routine and what is not is fuzzy and depends on each student's experience, because knowledge emerges from mathematical practices. Moreover, an important part of this knowledge remains tightly attached to the precise context of these practices. What takes the status of what we call in French *savoir* (knowledge) in order to differentiate it from *connaissance* (acquaintance), and is thus partially decontextualized is only a small part of what we know [Brousseau 1997]. What can be discovered through an assessment instrument is thus strongly dependent on the nature of the instrument, and on the way this instrument is situated with respect to the mathematical practices of the student who takes this test. What a test reveals is also very partial, a small part of what might be examined. The characterization of stuent competencies is created by the instrument. Another instrument that appears very similar might give very different results.

The theoretical model of Grugeon mentioned previously had four main dimensions: formation and treatment of algebraic objects, conversions between the language of algebraic representation and other semiotic registers (see, e.g., [Artigue 1999; Duval 1996]), the tool dimension of algebra, and the nature of the argumentation used in algebra. To each dimension were attached several subdimensions and for each of these, different criteria. In order to determine coherences in the algebraic functioning of the students, it was decided to test each criterion at least five times in different contexts. This resulted in an complex interrelated set of tasks: an approximately two-hour test was necessary just to get a workable account of the students' relationship with elementary algebra! This is the reason why we turn today towards technology, with the expectation that, thanks to the potential it offers, we could design a dynamic test where the tasks given to the students would be, after a first exploration of their cognitive state, carefully chosen in order to test and improve provisional models of their understanding of algebra, up to a reasonably stable level. Anyone who attended the workshop certainly remembers Deborah Ball's one and a half hour interview of the grade 6 boy (see Chapter 15), and can easily imagine how difficult it would be gain comparable insight into this boy's understanding of fractions using a written test, and how very different the images of his understanding might be if he were given different written tests. Despite the length of the interview, Ball mainly approached the object rather than the tool dimension of fractions, making the image we got rather partial.

Up to now, in this conclusion, I have focused on the difficulties we have approaching students' mathematical knowledge due to the diversity of its forms, due to its dependence on particular practices and contexts. This certainly must make us modest when approaching assessment issues. This also shows the importance of research. Thanks to educational research on algebra and fractions — not to mention other domains — we have at our disposal ways to approach the complex structure of students' mathematical knowledge in particular areas, for drawing inferences from what we observe, and for synthesizing the partial results we obtain in a productive way.

Nevertheless, despite the limitations of the enterprise that have been indicated, assessment is necessary to the life of educational systems, to the point that any form of knowledge which is not assessed or cannot be assessed lacks legitimacy. Approaching assessment within this perspective changes the general frame. From a cognitive frame we pass to an anthropological one, considering the role assessment plays in a human organization and questioning its ecology. We come thus to the second question: how to make assessment reflect the values we want to transmit through mathematics education? At the workshop, no one could escape the feeling that, in many U.S. states, the high-stakes tests that have been introduced do not reflect these values. Their negative effects were stressed by many speakers: as a result of testing the teaching enterprise was reduced to test preparation, and whatever the quality of these tests, this reduction had dramatic effects, For example, innovative experiments could no longer survive because they followed a completely different approach. On the other hand, those involved in the design of the tests showed the complexity of the work that is being carried out, expressing themselves in terms of validity, reliability, measurement, and scientific rigor. It was quite interesting to try to understand these discussions from the viewpoint of another culture with a different experience of assessment. In regard to the values that mathematics education intended to transmit, there is no distance at all: the discourse was the current discourse of democratic and developed countries who have to face immigration and increasing social inequalities.

Conversely, the educational culture with regard to assessment is rather different. In France, as described above, a national external examination at the end of high school is part of the culture. This examination, as I have tried to show, has to be aligned with the curriculum and also to respect a certain level of mathematical performance. Making these change is not easy. Many questions can be considered as routine questions. This is not, *per se*, a problem, because one of the goals of this examination is to check that some tasks have really become routine and that students have mastered the techniques they have been taught for solving them. What has been more closely scrutinized is the balance between routine and nonroutine tasks. I may be wrong but I do not have the feeling that the scientific rigor of the assessment is a core question. What seems to be important is to have — a utopia — all the students treated the same way thanks to the same national scale. That is, equity issues are paramount. And, for the government, what is important is to mantain the current pass rates on the examination. Nevertheless, the question of the influence of the *Baccalauréat* on teaching practices is also recognized as an important issue. Even on an examination with constructed-response questions, where students are asked to justify their answers, teaching practices excessively oriented towards its preparation limit the kind of tasks given to students, and above all, tend to reduce the diversity of students' modes of expression. For example, standard sentences are learned for justifying a given property. This has negative effects on the image of mathematics that students develop and does not help them make sense of mathematical practices — but for teachers and students this formulaic approach acts as an institutional norm. Some recent changes in the calculus syllabus have been introduced explicitly in order to avoid what have become template questions but there is no doubt that schools will quickly adapt to these changes.

Limiting the unavoidable negative effects from assessment and enhancing potential positive effects is not easy for educational systems. Doing so requires a better understanding of the effects of assessment and of the mechanisms that produce them, and it requires adequate systems of regulations grounded in those understandings. Further research is necessary in order to inform political decisions.

Appendix 1: Baccalauréat Test for Orientation S with Mathematics Speciality Given in Continental France in 2003

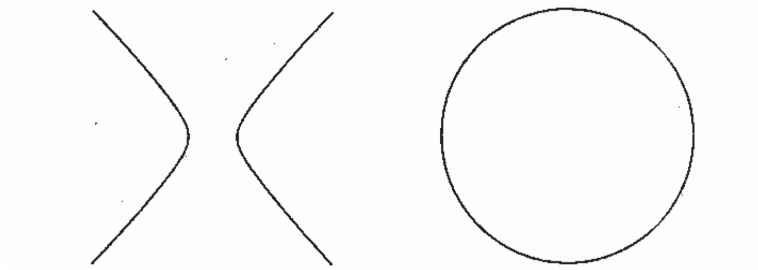
Exercise 1 (4 points)
In the complex plane with an orthonormal coordinate system (with unit length 2 cm), consider the points A, B and C whose respective affixes[12] are $a = 2$, $b = 1 - i$, and $c = 1 + i$.

1. (1 point)
   a. Place the points A, B and C on a figure.
   b. Compute $(c - a)/(b - a)$. Deduce from this computation that ABC is a right isosceles triangle.
2. (1 point)
   a. r is the rotation with center A such that r(B) = C. Determine the angle of r and compute the affix d of point D = r(C).
   b. Let Γ be the circle with diameter BC.
   Determine and construct the image Γ' of Γ under r.
3. (2 points)
   Let M be a point of Γ with affix z, distinct from C, and let M′ with affix z' be its image under r.
   a. Show that there exists a real number θ belonging to $[0, \pi/2) \cup (\pi/2, 2\pi)$ such that $z = 1 + e^{i\theta}$.
   b. Express z' as a function of θ.
   c. Show that $(z' - c)/(z - c)$ is a real number. Deduce from this that the points C, M, and M′ are on the same line.
   d. Place on the figure the point M with affix 1 + $e^{i2\pi/3}$ and construct its image M′ under r.

Exercise 2 (speciality exercise, 5 points)
Questions 3 and 4 are independent of Questions 1 and 2, except that the equation Γ given in Question 1c occurs in Question 4.

[12] A point in the complex plane with cartesian coordinates (a, b) is said to have affix $a + bi$.

1. (1.5 points)
   The space is equipped with an orthonormal system of coordinates.
   a. Show that the planes P and Q whose respective equations are:
   $$x + \sqrt{3}y - 2z = 0 \quad \text{and} \quad 2x - z = 0$$
   are not parallel.
   b. Give a parametric representation for the line Δ, the intersection of the planes P and Q.
   c. Consider the cone of revolution Γ whose axis is the x-axis and has the line Δ as a generator.
   Prove that the equation of Γ is $y^2 + z^2 = 7x^2$.
2. (1 point)
   Two intersections of Γ with planes, each plane parallel to a coordinate axis, are represented below.
   In each case, determine a possible equation of the plane, and carefully justify your answer.



3. (1.5 points)
   a. Show that the equation $x^2 \equiv 3 \pmod 7$, whose unknown x is an integer, does not have solutions.
   b. Prove the following property:
   For every integer a and b, if 7 divides $a^2 + b^2$, then 7 divides a and 7 divides b.
4. (1 point)
   a. Let a, b and c be integers different from 0. Prove the following property: if point A with coordinates (a, b, c) is on the cone Γ, then a, b, and c are multiples of 7.
   b. Deduce from it that the only point of Γ whose coordinates are integers is the summit of this cone.

Problem (11 points)
Let N_0 be the number of bacteria introduced in a growth medium at time $t = 0$ (N_0 is a positive real number in millions of bacteria).
This problem aims to study two growth models for this bacterial population:

A first model for a short period following the introduction of the bacteria into the growth medium (Part A).

A second model which can be used over a longer period (Part B).

Part A (2 points). In the time following the introduction of the bacteria into the growth medium, assume that the rate of increase of the bacteria is proportional to the number of bacteria.

In this first model, denote by $f(t)$ the number of bacteria at time t (expressed in millions of bacteria). The function f is thus a solution of the differential equation $y' = ay$ (where a is a positive real number depending on the experimental conditions).

1. Solve this differential equation with the initial condition $f(0) = N_0$.
2. T is the time of doubling of the population. Prove that, for every positive number t:
$$f(t) = N_0 2^{(t/2T)}$$

Part B (3 + 4 points). The medium being limited (in volume, in nutritive elements . . .), the number of bacteria cannot indefinitely increase in an exponential way. The previous model cannot thus be applied over a long period. In order to take these observations into account, the increase in the population of bacteria is represented as follows.

Let $g(t)$ be the number of bacteria at time t (expressed in millions of bacteria), g is a strictly increasing function which admits a derivative on $[0, \infty)$, and satisfies on this interval the relationship:

$$\text{(E)} \qquad g'(t) = ag(t)\left[1 - g(t)/M\right]$$

where M is a positive constant which depends on the experimental conditions and a is the real number given in Part A.

1. a. Prove that if g is a strictly positive function satisfying (E) then the function $1/g$ is a solution of the differential equation (E$'$): $y' + ay = a/M$.
   b. Solve (E$'$).
   c. Prove that if h is a strictly positive solution of (E$'$) $1/h$ satisfies (E).
2. In the following, it is assumed that, for every positive real number t, we have $g(t) = M/(1 + Ce^{-at})$, where C is a constant, strictly larger than 1, dependent on the experimental conditions.

a. Find the limit of g at ∞ and prove that, for every nonnegative real number, the two inequalities $0 < g(t) < M$ hold.

b. Study the variations of g (it is possible to use the relation (E)).
Prove that there exists a unique real number t_0 such that $g(t_0) = M/2$.

c. Prove that $g'' = a(1 - 2g/M)g'$. Study the sign of g''. Deduce from this that the rate of increase of the number of bacteria is decreasing after the time t_0 defined above. Express t_0 as a function of a and C.

d. Knowing that the number of bacteria at time t is $g(t)$, calculate the mean of the number of bacteria between time 0 and time t_0, as a function of M and C.

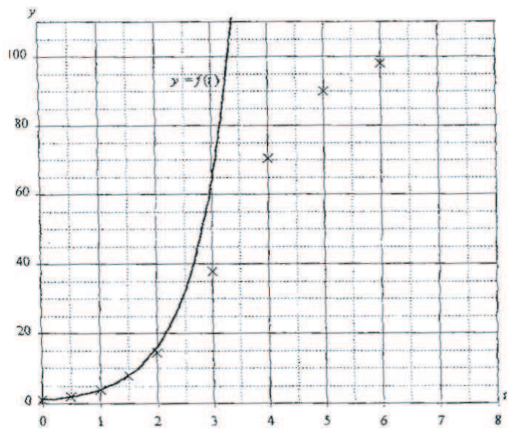
Part C (2 points)

1. The table below shows that the curve representing f passes through the points with coordinates $(0, 1)$ and $(0.5, 2)$.
   Deduce from that the values of N_0, T and a.
2. Knowing that $g(0) = N_0$ and that $M = 100N_0$, prove that for every positive real number the following equality holds:

$$g(t) = 100/(1 + 99 \times 4^{-t}).$$

3. Draw on the attached sheet [see graph below] the curve Γ representing g, the asymptote to Γ and the point of Γ with abscissa t_0.
4. Under what conditions does the first model seem to you consistent with the observations made?

t (in h)	0	0.5	1	1.5	2	3	4	5	6
Millions of bacteria	1.0	2.0	3.9	7.9	14.5	37.9	70.4	90.1	98



Appendix 2: New Proposals For Assessment at the Baccalauréat

Exercise 6 The plane is equipped with an orthonormal system of coordinates. Let f be the function defined on $\mathbb{R}$ by

$$f(x) = e^{2x}/2 - 2.1e^{x} + 1.1x + 1.6.$$

1. Graph the curve representing f in the window $-5 \leq x \leq 4$, $-4 \leq y \leq 4$. Reproduce the curve on your assessment sheet.
2. From this graphic representation, what do you conjecture:
   a. About the variation of the function f (where f is positive or negative, increases or decreases).
   b. About the number of solutions of the equation $f(x) = 0$.
3. Now we want to study the function f.
   a. Solve in $\mathbb{R}$ the inequality $e^{2x} - 2.1e^{x} + 1.1 \geq a$.
   b. Study the variation of f.
   c. Deduce from that study the number of solutions of the equation $f(x) = 0$.
4. If you want to draw the curve of f on the interval $[-0.05, 0.15]$ and to visualize the results of Question 3, what extreme values for y can be chosen for the calculator window?

Exercise 7 Consider a sequence $\{u_n\}$ with $u_n \geq 0$, and the sequence $\{v_n\}$ defined by

$$v_n = u_n/(1 + u_n).$$

Are the following propositions true or false? Justify your answer in each case.

1. For every n we have $0 \leq v_n \leq 1$.
2. If the sequence $\{u_n\}$ is convergent, so is the sequence $\{v_n\}$.
3. If the sequence $\{u_n\}$ is increasing, so is the sequence $\{v_n\}$.
4. If the sequence $\{v_n\}$ is convergent, so is the sequence $\{u_n\}$.

Exercise 20 The different questions are independent. All the answers have to be carefully justified.

1. In each of the following cases, give a function f satisfying the given properties. An expression for f must be given.
   a. f is defined on $\mathbb{R}$ by $f(x) = ae^{2x} + be^{x} + c$, the limit of f at ∞ is ∞, and the equation $f(x) = 0$ has two solutions, 0 and ln 2.
   b. f is defined on $[0, \infty)$, $f(2) = 4$ and, for every positive[13] real numbers x and y, we have $f(xy) = f(x) + f(y)$.

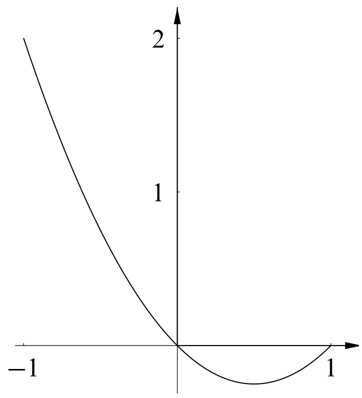
[13] Here and below "positive" is used with its usual English meaning, so in this case $x > 0$ and $y > 0$.

c. f is a polynomial function of degree at least 2 and the mean value of f on $[-2, 2]$ is 0.

2. Let g be a function that is defined and differentiable with derivative g' that is continuous on $[-1, 1]$. The curve representing g is drawn below.
Are the following assertions consistent with this graph?

a. $\int_{-1}^{1} g'(x)\, dx = 0$

b. $\int_{-1}^{1} g(x)\, dx > -\frac{1}{2}$



Exercise 30 Strange numbers!
Numbers such as 1, 11, 111, 1111, etc. are called rep-units. They are written only with the digit 1. They have a lot of properties which intrigue mathematicians. This exercise has the aim of revealing some of these.

For a positive integer k, the rep-unit written with k 1's is denoted N_k:

$$N_1 = 1, \quad N_2 = 11, \ldots$$

1. Give two prime numbers less than 10 that never appear in the decomposition in prime factors of a rep-unit. Justify your answer.
2. Give the decomposition in prime factors of N_3, N_4, N_5.
3. Let n be an integer strictly larger than 1. Suppose that the decimal expression of n^2 ends in 1.

a. Prove that the decimal expression of n ends in 1 or in 9.

b. Prove that there exists an integer m such that n can be written in the form $10m + 1$ or $10m - 1$.

c. Deduce from this that $n^2 \equiv 1 \pmod{20}$.

4. a. Let $k \geq 2$. What is the remainder in the division of N_k by 20?

b. Deduce from this that a rep-unit different from 1 cannot be a square.

Appendix 3: 2002 Grade 6 Diagnostic Test Exercises with Allocated Time and Pass Rates

NUMBERS AND OPERATIONS

Exercise 1 (2.5 min) *Mental calculation*

In box a, write the result of: 198 plus 10	84.1%
In box b, write the result of: 123 plus 2 tens	73.8%
In box c, write the result of: 37 divided by 10	56.0%
In box d, write the result of: 7 multiplied by 10,000	87.9%
In box e, write the result of: 405 minus 10	80.8%

Exercise 5 (2 min) 86.5%

A bus started from the school at 8:30 a.m. and arrives at the museum at 9:15 a.m. How long did it take to travel?

Exercise 12 (2 min) 72.4%

Order the following numbers from smallest to largest.

2 2.02 22.2 22.02 20.02 0.22

$\ldots\ldots < \ldots\ldots < \ldots\ldots < \ldots\ldots < \ldots\ldots < \ldots\ldots$

Exercise 13 (2 min) 57.6%

Here are five numbers ordered from smallest to largest.
Write the number 3.1 in the correct place.

. . . . . . 2.93 . . . . . . 3 . . . . . . 3.07 . . . . . . 3.15 . . . . . . 3.4

Exercise 18 (3 min)

Calculate:

a) $58 - (8 + 22) =$	73.7%
b) $5 \times (9 - 6) =$	82.0%
c) $(7 + 13)$ x $3 =$	86.6%
d) $15 \div (2 + 3) =$	68.2%

Exercise 21 (3 min)

Write the calculations in the boxes provided: [boxes omitted]

a. 8.32 + 15.87	84.6%	b. 15.672 + 352.21	79.2%

PROBLEMS

Exercise 10 (3 min) 48.0%

A school has two classes. In the school there are 26 girls. In the first class, there are 12 girls and 11 boys. In the second class, there are 27 pupils. How many boys are there in the second class?

Exercise 31 (4 min 30 sec)

A pack of mineral water is made of 6 bottles of 1.5 liters.
A shopkeeper arranges 25 such packs.

a. Circle the calculation that yields the number of bottles arranged. 81.4%

$25 + 6$	25×6	1.5×6
$25 - 6$	25×1.5	$6 - 1.5$

b. The shopkeeper made the calculation $(25 \times 6) \times 1.5$.
The result of this calculation is 225.
What quantity did the shopkeeper want to know? 59.1%

Exercise 14 (4 min)

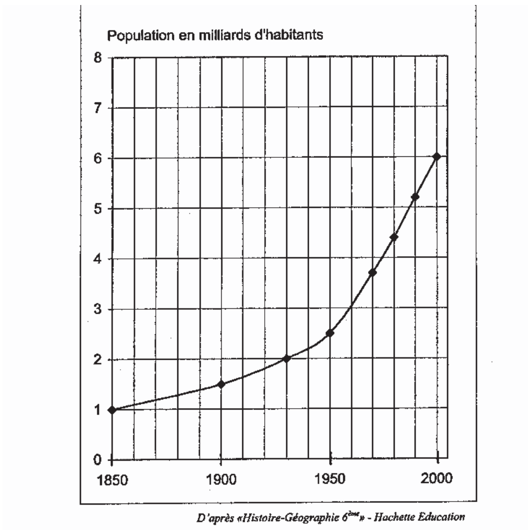
The following graph represents the increase in the world population since 1850 (in billions).

a. In what year did the world population reach 2 billion? 78.4%

b. What was the world population in 2000? 86.1%

c. How many years were necessary to go from 1 to 2 billion? 40.1%

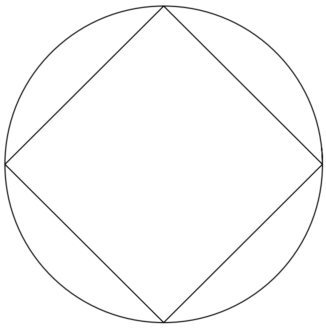
d. Approximately what was the world population in 1950? 53.2%

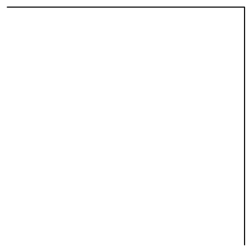


GEOMETRY

Exercise 24 (4 min) 95.7%

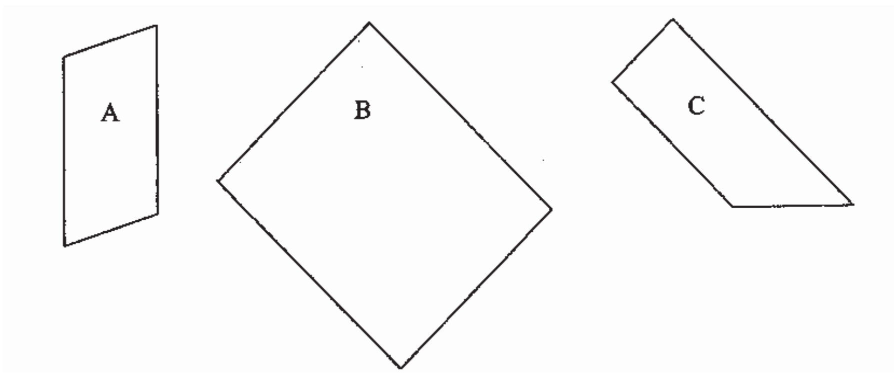
Below, on the left, is a figure made of a square and a circle.
Somebody began to reproduce this figure. Two sides of the square are already drawn. Complete the drawing.





Exercise 6 (5 min)

Here are three figures.



Fill in the table.

Figure	Is it a rectangle ? (Circle the answer.)	Explain how you found it.
A	YES NO	42.9%
B	YES NO	37.2% + 45.9%
C	YES NO	39.9% + 47.9%

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