

The seminal formula of Gross and Zagier relating heights of Heegner points to derivatives of the associated Rankin  $L$ -series has led to many generalizations and extensions in a variety of different directions, spawning a fertile area of study that remains active to this day.

This volume, based on a workshop on Special Values of Rankin  $L$ -Series held at the MSRI in December 2001, is a collection of articles written by many of the leading contributors in the field, having the Gross–Zagier formula and its avatars as a common unifying theme. It serves as a valuable reference for mathematicians wishing to become better acquainted with the theory of complex multiplication, automorphic forms, the Rankin–Selberg method, arithmetic intersection theory, Iwasawa theory, and other topics related to the Gross–Zagier formula.



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Heegner Points and Rankin  $L$ -Series

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