Hopf algebras have important connections to quantum theory, Lie algebras, knot and braid theory, operator algebras, and other areas of physics and mathematics. They have been intensely studied in the last decade; in particular, the solution of a number of conjectures of Kaplansky from the 1970s has led to progress on the classification of semisimple Hopf algebras and on the structure of pointed Hopf algebras. There has been much progress also on actions and coactions of Hopf algebras and on Hopf Galois extensions. Many new methods have been used for these results: modular and braided categories, representation theory, algebraic geometry, and Lie methods such as Cartan matrices.

The contributors to this volume of expository papers were participants in the Hopf Algebras Workshop held at MSRI as part of the 1999–2000 Year on Noncommutative Algebra. Together the papers give a clear picture of the current trends in this active field, with a focus on what is likely to be important in future research.

Among the topics covered are results toward the classification of finitedimensional Hopf algebras (semisimple and non-semisimple), as well as what is known about the extension theory of Hopf algebras. Some papers consider Hopf versions of classical topics, such as the Brauer group, while others are closer to recent work in quantum groups. The book also explores the connections and applications of Hopf algebras to other fields.

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