

Model theory is a branch of mathematical logic that has found applications in several areas of algebra and geometry. It provides a unifying framework for the understanding of old results and more recently has led to significant new results, such as a proof of the Mordell–Lang conjecture for function fields in positive characteristic. Perhaps surprisingly, it is sometimes the most abstract aspects of model theory that are relevant to these applications.

This book gives the necessary background for understanding both the model theory and the mathematics behind the applications. Aimed at graduate students and researchers, it is unique in that it contains surveys by leading experts covering the whole spectrum of contemporary model theory (stability, simplicity, o-minimality and variations), and introducing and discussing the diverse areas of geometry (algebraic, diophantine, real analytic,  $p$ -adic and rigid) to which the model theory is applied.

The book begins with an introduction to model theory by David Marker. It then broadens into three components: pure model theory (Bradd Hart, Dugald Macpherson), geometry (Barry Mazur, Ed Bierstone and Pierre Milman, Jan Denef), and the model theory of fields (Marker, Lou van den Dries, Zoé Chatzidakis).



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Model Theory, Algebra, and Geometry

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