Several Complex Variables
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Preface

This volume consists of sixteen articles written by participants of the 1995–96 Special Year in Several Complex Variables held at the Mathematical Sciences Research Institute in Berkeley, California.

The field of Several Complex Variables is a central area of mathematics with strong interactions with partial differential equations, algebraic geometry and differential geometry. The 1995–96 MSRI program on Several Complex Variables emphasized these interactions and concentrated on developments and problems of current interest that capitalize on this interplay of ideas and techniques.

This collection provides a picture of the status of research in these overlapping areas at the time of the conference, with some updates. It will serve as a basis for continued contributions from researchers and as an introduction for students. Most of the articles are surveys or expositions of results and techniques from these overlapping areas in several complex variables, often summarizing a vast amount of literature from a unified point of view. A few articles are more oriented toward researchers but nonetheless have expository sections.

On August 29, 1997 Michael Schneider, one of the two editors of this volume, died in a rock-climbing accident in the French Alps. This volume is dedicated to his memory. The front matter includes his portrait, a listing of the major events in his mathematical career, and a selection of his mathematical contributions.

Michael Schneider

May 18, 1942 to August 29, 1997

- Studied mathematics and physics with O. Forster and K. Stein at the University of Munich, with one semester in Geneva.
- Diploma, University of Munich, 1966.
- Doctorate, University of Munich, 1969, with a dissertation on complete intersections in Stein manifolds.
- Assistant, University of Regensburg, 1969–1974.
- Habilitation, University of Regensburg, 1974.
- Professor, University of Göttingen, 1975–1980.
- Chaired Professor, University of Bayreuth, 1980–1997.
- Editor, *Journal für die reine und angewandte Mathematik*, 1984–1995.
- Served as Fachgutachter of the Deutsche Forschungsgemeinschaft and on the Wissenschaftlichen Beirat of the Forschungszentrum Oberwolfach.

Selected Mathematical Contributions

- A complex submanifold Y of a Stein manifold X with $\dim Y = \frac{1}{2}\dim X$ is Stein if and only if the normal bundle of Y is trivial and the fundamental class defined by Y vanishes as a homology class in X.
- If the normal bundle of a complex submanifold Y of codimension k in a compact complex manifold X is positive in the sense of Griffiths, then $X - Y$ is k-convex.
- (with Badescu) If the normal bundle of a d-dimensional complex submanifold Y in a projective manifold X is $(d-1)$-ample in the sense of Sommese, then the field of formal meromorphic functions along Y is a finite field extension of the field of meromorphic functions on X.
- A stable vector bundle of rank 2 on $\mathbb{P}_n$ is ample if and only if $c_1 \geq 2c_2 - \frac{c_1^2}{2}$.
- (with Elencwajg and Hirschowitz) If E is a holomorphic vector bundle of rank $r \leq n$ on $\mathbb{P}_n$ which has the same splitting type on every line, then E either is split or is isomorphic to $\Omega^1_{\mathbb{P}_n}(a)$ or $T_{\mathbb{P}_n}(b)$.
- For a stable vector bundle of rank 3 on $\mathbb{P}_n$ with $c_1 = 0$ one has $|c_3| \leq c_2^2 + c_2$.
- For a semistable bundle E of rank 3 on $\mathbb{P}_n$ $(n \geq 3)$ and a general hyperplane H in $\mathbb{P}_n$, the restriction $E|H$ is semistable unless $n = 3$ and E is a twist of the tangent or cotangent bundle of $\mathbb{P}_3$.
- (with Catanese) If X is an n-dimensional Cohen–Macaulay projective variety which is nonsingular outside a subvariety of codimension at least 2 and H is a very ample divisor in X and E is a vector bundle on X, then there exists a polynomial function $P_{n,h,E}$ in the first h Chern classes of E and the first two Chern classes of X such that for every nonzero section of E whose scheme of zeroes Z has codimension h, one has $\deg(Z) \leq P_{n,h,E}$.

- (with Beltrametti and Sommese) Complete classification of all threefolds of degree 9, 10, and 11 in $\mathbb{P}_5$.
- (with Braun, Ottaviani and Schreyer) There are only finitely many families of threefolds in $\mathbb{P}_5$ which are not of general type.
- There are only finitely many families of m-dimensional submanifolds in $\mathbb{P}_n$ not of general type if $m \geq \frac{n+2}{2}$.
- An n-dimensional compact complex manifold with ample cotangent bundle cannot be embedded into $\mathbb{P}_{2n-1}$.
- (with Demailly and Peternell) If X is a compact Kähler manifold whose tangent bundle is numerically effective, then the Albanese map of X is a surjective submersion and, after possibly replacing X by a finite étale cover, the fibers of the Albanese map of X are Fano manifolds with numerically effective tangent bundles and the fundamental group of X agrees with that of its Albanese.
- (with Demailly and Peternell) Let X be a compact Kähler manifold with numerically effective anticanonical line bundle. If the anticanonical line bundle of X admits a metric with semipositive curvature, then the universal cover of X is a product whose factors are either Euclidean complex spaces, Calabi–Yau manifolds, symplectic manifolds, or manifolds with the property that no positive tensor powers of the cotangent bundle admit a nonzero holomorphic section. In particular, the fundamental group of X has an abelian subgroup of finite index. If the anticanonical line bundle of X is only numerically effective, then the fundamental group of X has subexponential growth and, in particular, X does not admit a map onto a curve of genus at least 2.
- (with Barlet and Peternell) If X is a $\mathbb{P}_2$-bundle over a compact algebraic surface, then any two nonsingular surfaces with Griffiths-positive normal bundle in X must intersect.
- (with Peternell) Let X be a compact complex threefold and Y be a complex hypersurface in X whose complement is biholomorphic to $\mathbb{C}^3$. Then X is projective if Y is normal or if the algebraic dimension $a(X)$ of X is 2, or if $a(Y) = 2$. Moreover, the cases $a(X) = 2$ or $a(X) = 1$ with a meromorphic function on X whose pole-set is Y cannot occur.
- (with Catanese) Let X be a projective n-dimensional manifold of general type and G be an abelian group of birational automorphisms of X. If the canonical line bundle K_X of X is numerically effective, then the order of G is bounded by $C_n(K_X^n)^{2^n}$, where C_n is a constant depending only on n. If $n = 3$ and if m is a positive integer admitting a G-eigenspace in $H^0(X, mK_X)$ of dimension at least 2, then the order of G is bounded by the maximum of $6P_2(X)$ and $P_{3m+2}(X)$, where $P_q(X)$ is the dimension of $H^0(X, qK_X)$.