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# Redesigning the Calculus Sequence at a Research University

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Abstract. The School of Mathematics at the University of Minnesota is developing a new calculus sequence for students in mathematics, science, and engineering. The sequence incorporates changes in content and methods of instruction. The students are routinely asked to work together cooperatively in small groups. An innovative feature involves student exploration of mathematical ideas and complex, open-ended, interdisciplinary applications using interactive features of the World Wide Web. This paper describes some of the special features of the sequence, including some different approaches to instructional teamwork and student-centered instruction. Student attitudes about the usefulness of the pedagogical and curricular components, and how these approaches affect their learning, are analyzed for this new sequence. Quantitative data are presented that compare the achievement and retention of students in the new sequence with a control group from the standard calculus sequence. Future directions for refinements in teaching the sequence, new curricular approaches, and additional statistical information are discussed.

## Introduction

The major goal of the University of Minnesota Calculus Initiative is to create a challenging sequence for mathematics, science, and engineering students in which students obtain a better understanding of how to use calculus as a tool both for mathematical analysis and for solving problems in other disciplines. The new sequence is primarily intended for the middle 67% of the calculus-ready science and engineering students, and for some social science majors. The Initiative is a major reconceptualization of the traditional large lecture/recitation approach used in many doctoral institutions, and incorporates key aspects of successful calculus reform efforts in a new instructional format and supportive learning environment. This reorganization includes changes in curriculum, content, and methods of instruction supported by the use of technologies, and in pedagogy that encourages student-centered learning by increasing student/faculty interaction.

Interdisciplinary or applications-based projects and labs are a central component of the Initiative. They engage students in the analysis of science or engineering-based problems that interlace mathematical concepts with important scientific and engineering applications. Student teams investigate these applications in a workshop setting. The teams collect and analyze data, formulate and test conjectures, and communicate their ideas through oral and written reports. These investigations require students to become more actively involved in the learning process and help them to meet the increased responsibility for acquiring routine computational skills. Improvements in student attitudes about mathematics and the use and value of mathematical reasoning in other disciplines are anticipated outcomes. The materials have been designed so that strongly motivated students can remain challenged and able to explore topics in great depth, while less well prepared students can focus on the mastery of essential concepts. All layers of the materials foster teamwork and stress the importance of clear communication of mathematical ideas. Ideas for the applications-based projects were developed through discussions with science and engineering faculty. The content and technical aspects of the projects were created as a cooperative effort with the Geometry Center. The technologies used vary as the sequence progresses. In the first year, students extensively use graphing calculators with a few forays into labs that utilize interactive hypertext programs that run over the Internet. During the second year, students use Maple and Matlab in regular laboratory settings, although the essential components of these materials can be made accessible to classrooms whose technology is limited to graphing calculators.

In principle, the new calculus sequence can be implemented with minimal resources beyond the traditional large lecture model (explained below). The current model has 100 students per professor, but the students meet in small classes of 25 during 3 of the 5 hours each week. The workshop setting has created excellent opportunities for extensive small-group activities and experiences, and provides the overall atmosphere of a small classroom. The additional instructional cost has been relatively modest, approximately $225 per year per student.

This paper provides a careful analysis of the quantitative and qualitative student outcomes that the new sequence hopes to achieve. Outcomes are compared with a control group of 240 mathematics, science, and engineering students with similar placement scores who participated in the standard calculus sequence. Quantitative data about retention, achievement, and successful completion of the entire first-year calculus sequence is developed for both groups. Qualitative data about student attitudes regarding the usefulness of the structure of the sequence, pedagogical approaches, and the content of the sequence are examined for the Initiative students. This includes initial positive results about homework and group work as enhancements to learning.

## Implementation

The curricular and structural aspects of the Initiative are described in this section.

**Curriculum and Expectations.** The primary goal is to have the students leave calculus with the mathematical skills they need to succeed in future science and engineering courses, and ultimately in their careers. Secondary goals are (1) to motivate students to remain in the calculus sequence and in the Institute of Technology (IT), (2) to encourage the students to become more responsible for their learning, and (3) to help the students understand the usefulness of calculus. The six distinct features of the curriculum that address these goals are described below:

1. *Mandatory Computational Proficiency.* Students may not receive a course grade until they have successfully passed a gateway exam, computing 8 of 10 standard derivatives (first quarter) and integrals (second quarter).
2. *Type of Text.* The text is A. Ostebee and P. Zorn, "Calculus from Graphical Numerical and Symbolic Points of View," a reform text emphasizing geometric reasoning, visualization, and conceptual approaches and written at a level that students can read and understand before going to lecture.
3. *Emphasis on Concepts, Geometric Reasoning, and Visualization.* Students are required to go beyond algebraic manipulations, and use geometric and visual reasoning to analyze complex functions and functions defined solely by graphs. This expectation of conceptual understanding is reflected in the examinations, the homework, and the projects.
4. *Communication of Mathematical Ideas.* Students are expected to present mathematics professionally, both orally and in writing. They give solutions in class, write up certain homework problems using complete sentences, clear exposition, graphs, and diagrams to support the solution, and develop project reports each quarter.
5. *Integration of Applications.* Real-life problems and examples are central to the course, in contrast to the traditional approach of giving special "applications problems." Application modules include topics from: optics (mathematics and the geometry of rainbows), limnology (analysis of $CO_2$ production in fresh water systems), civil and mechanical engineering (static deformation of cantilevered and simply-supported beams), and biology and ecology (population dynamics using dynamical systems).
6. *Problem-Solving.* While template and drill problems are used to develop skills, the majority of the problems for students require a synthesis of the ideas con-

tained in the course. The homework, tests, and quizzes emphasize synthesis problems.

**Structure.** The four main components of the new course are the lecture, the workshops, the lab, and the homework. These have been designed to reinforce each other, and to encourage and support active learning.

*Format of Lecture.* Twice a week, the students attend a one hour large class session conducted by the professor. The main purpose of these sessions is to present new concepts, to provide mathematical explanations of the concepts introduced in the workshop the previous day, and to provide guided practice for homework problems.

*Format of Workshops.* The students attend one 50-minute workshop and one 100-minute workshop per week; these workshops have been designed by a lead workshop instructor to ensure continuity across the sections. The course professor or workshop leader assists the instructor during the 100 minute workshop. One workshop features problems at the average level of the homework problems, allowing the students time for guided practice. Activities in the other workshop foreshadow future concepts, allowing the students the opportunity to develop computational expertise for upcoming theory. Important aspects of both types of activities are effective communication and the development of problem-solving abilities with peers.

*Format of Labs.* Four times during the first year, each student works—in a team of three or four members—through large scale applications using the mathematics he/she has learned. Although the students spend only a few class sessions working with their teams, the labs are available over the World Wide Web, allowing the students to continue out-of-class work from their home, dorm room, or a campus terminal. Initially, hard copies are distributed in class; these are followed by a workshop that meets in a computer lab. In the lab, students run the experiments and simulations and get assistance as needed. Each team develops a lab report over the next few weeks outside of class in which ten to twenty extended questions are addressed. In addition to learning some non-trivial mathematics, the students practice being part of a research team: allocating efforts, overseeing quality, making design decisions, and communicating final results.

## Findings

To substantiate which student outcomes were influenced by the pilot sequence curriculum and pedagogy, a control group of 240 students was created from the IT students enrolled in the standard calculus sequence. For sampling consistency, the control group students were selected primarily on the basis of their placement exam scores and secondarily on their status as incoming Institute of Technology freshmen. A database was developed to track students in the control group throughout the standard calculus sequence and their upper division engineering and science course work. A parallel database has also been created for the pilot sequence students. While the overall placement score pattern is similar, the cohort distributions do indicate some differences. Thus, direct comparisons may become more reliable when looking at sub-cohorts (e.g., all students with a particular placement exam score) than across the entire population.

**Achievement.** The percentage of students in the pilot group who successfully completed (achieved a final grade of "C" or higher) each quarter of the first year of the sequence averaged 97% compared to the control group average of 85%. Especially significant was the grading process and the number of high grades ("A"s). The pilot sequence set higher expectations for work, conceptual understanding, and technological proficiency for its students than did the standard sequence, and at least initially demanded much more time and effort. In the first quarter, student adjustments to this increased set of expectations and some unrealistic homework expectations led to a slightly elevated grading process, noting that certain aspects of the student performance (attendance and class participation) exceeded initial expectations and had a positive influence on the grades. In the second and third quarters, as students and faculty expectations stabilized, grading became more standardized, and grades more carefully reflected high expectations and achievement. Thus it is significant that the percentage of "A"s in the pilot courses remained constant at 42-43% throughout the first-year sequence. This average high success rate of 43% is significantly higher than the control group average of 22%. Assigning grade values of $A = 4$, $B = 3$, and $C = 2$, the average grade point average (GPA) for students in the pilot course was 3.31, a difference of .43 compared to an average of 2.88 in the control group, or nearly half a grade level higher in average achievement.

For overall completion, 66% of the students initially in the pilot course successfully completed the pilot sequence versus 62% of the control group students in the standard sequence. More significant was the percentage of students (83% of 97) who successfully completed the Fall quarter of the pilot sequence and then successfully completed a year of calculus (in either the pilot or the standard sequence). In comparison, of the 240 control group students, only 62% successfully completed the first year of calculus.

**Retention.** Eighty percent of the students who initially enrolled in the pilot sequence have registered for a second year of calculus either in the pilot or in the standard sequence compared to 61% of the control group students. An interesting trend is the consistently increasing quarterly retention rate of the pilot group students—80% of the students who started in the pilot sequence enrolled in the second quarter pilot course, 87% of the students enrolled in the second quarter pilot course took the third quarter course, and 93% of the students enrolled in the third quarter of the pilot course registered for the second year of the pilot sequence; in contrast, retention of the control group students between quarters remained constant at 85%.

**Student Expectations and Reactions.** Student attitudes about the usefulness of the pedagogical and curricular components and how these approaches affected their learning were collected for the pilot sequence. Student ratings and comments on group work (peer collaboration) and homework were used to determine whether the students became more responsible for their learning, and student ratings and comments on the workshops and the lab projects were used to analyze students' understanding of the usefulness of calculus to science and engineering applications.

*Increased Student Responsibility for Learning.* An average of 88% of the students rated group work and peer collaboration as a useful-to-extremely-useful element of a calculus course, and 80% rated homework as a useful-to-extremely-useful course component on the survey given during orientation (one day prior to the start of the Fall 95 first quarter). Nearly a quarter of all of the students (24%) stated that group work or collaboration with fellow students was one of the most important elements of a calculus course, and 22% stated that homework assignments were one of the most important elements of a calculus course.

*Group Work.* On the first quarter mid-term survey, small-group work was rated as useful-to-extremely-useful by only 61% of the students. This increased to 64% on the second quarter mid-term survey and to 78% by the third-quarter mid-term survey, possibly reflecting the growing ability of students to productively collaborate in group settings. Collaboration with peers remained consistently rated at 76%, 74%, and 76% on the mid-term assessments. Students who agreed or strongly agreed with the statement: "Time spent working in groups was worthwhile" formed 60% of the total in the first quarter, 83% in the second quarter, and 89% in the third quarter; the corresponding statistics for the statement: "Working in small groups led to understanding of the subject matter" were 87% in the first quarter, stabilizing to 82-83% in the second and third quarters.

*Homework.* In the first quarter, the expectations for the homework assignments, including extensive professionally-written problem sets, approached honors-level and competed for study time with other challenging pre-engineering and science courses. Hence only 47% of the students rated the homework as useful-to-extremely-useful on the first quarter mid-term survey, down from 80% on the orientation survey. Adjustments by the instructional team to the assignment load resulted in improved homework quality and student attitudes in subsequent quarters. We noted that 83% of the students rated homework as useful to extremely useful on the second quarter mid-term survey and 76% of the students rated homework as useful-to-extremely-useful on the third quarter mid-term survey. Students who agreed or strongly agreed with: "The amount of homework was appropriate for the course" went from 48% to 81% at the end of the second quarter, and 91% at the end of the third quarter, while the corresponding ratings for "The grading of the homework reflected my understanding" went from 59% to an average of 80% by the end of the second and third quarters.

**Increased Student Understanding of the Usefulness of Calculus.** Student survey ratings and comments on the workshops and the lab projects were used to quantify the relationship between the students' understanding of the usefulness of calculus to science and engineering applications and the Initiative's curriculum and pedagogy.

*Workshops.* Students who agreed or strongly agreed with the statement: "The workshop classroom atmosphere was a positive learning environment" formed 88% of the total at the end of the first quarter, 90% at the end of the second quarter, and 95% at the end of the third quarter. The students also gave a high rating to their interaction with the workshop instructors: 94%–100% of the students agreed or strongly agreed that the availability, attitude, and support of the workshop instructors had a positive influence on their learning.

*Lab Projects.* The survey results suggest that the students believe that the extended projects were worthwhile and worked well for learning both mathematics and real-life applications; some, however, expressed concerns about the extra time required to complete these projects. Overall, 65% rated the lab project as a moderately-useful-to-useful course component. When asked if the lab project enhanced their understanding of calculus and its application, 38% of the students answered "yes", 24% answered "maybe", and 38% answered "no".

Although the students ranked the lab projects as useful overall throughout the sequence, specific items about the labs indicated some differences. While 47% agreed-to-strongly-agreed that time spent on the Numerical Integration Lab was worthwhile, only 36% did so for the Beam Lab. One possible explanation for the popularity of the Numerical Integration Lab was its use in place of a chapter from Ostebee and Zorn, making the reason for doing this project clearer to the students.

## The Future

Considering the success of the Initiative's pilot year, two large class sections of the first year sequence will be taught in 1996–97. One of the workshop subsections will be taught in a residential hall to IT freshmen who are part of the University's Residential College program. In addition, some selected liberal arts students with similar mathematics competence and interest as the IT students will be invited to participate.

Each section's instructional team consist of one faculty member, four graduate student workshop instructors, and a fifth graduate assistant in the new role of workshop leader. The faculty member will conduct the two large class sessions per week and rotate between two of the four workshop sessions. The workshop leaders will develop workshop materials and exams, and rotate between the other two workshops. The workshop instructors will conduct two workshops per week and assist in the large class sessions. The instructional teams will meet weekly to ensure consistency across all sections.

The second year sequence includes about 15% transfer students from first-year calculus courses similar to the pilot course, and strong students from the standard Minnesota calculus sequence in addition to students who completed the 1995–96 pilot sequence. Mathematics departments at local state and community colleges have been made aware of this opportunity.

The content of the new second year sequence is multivariable calculus and differential equations using elementary linear algebra, emphasizing geometric solutions of differential equations and building on the conceptual framework established during the first year. Concepts from linear algebra will be introduced as needed for the analysis of curves, surfaces, and systems of differential equations. Students will begin to use higher-level technological tools such as symbolic software and numerical software for linear algebra and dynamical systems. The students will engage in shorter but more frequent labs—4 or 5 per quarter—to explore applications which help students to visualize abstract ideas that arise in the geometry of curves and surfaces. By the end of the sequence, it is expected that the students will be comfortable using symbolic, visual, and numerical techniques to attack complex problems, and to understand when such techniques break down. The five quarter core sequence will prepare students to pursue several options for a sixth quarter, including topics such as complex analysis, vector field theory, or an introduction to numerical methods.

## Conclusions

The new calculus sequence for science and engineering students differs from the traditional sequence in content, method of instruction, and in the relative emphasis on group-work, interdisciplinary applications, and technology. The initial achievement and retention findings indicate the following:

(i) 83% of the pilot group students successfully completed the first year of calculus compared to 62% of the control group students,
(ii) the average GPA of successful completion for the pilot sequence students was 3.31 compared to 2.88 for the control group, and
(iii) 80% of the pilot group students are enrolled in a second year of calculus compared to 61% of the control group students.

Pilot group student reactions indicated the following:

(i) The usefulness of small group work (61% rated it as a useful-to-extremely-useful component mid-Fall 95, 64%, mid-Winter 96, and 78%, mid-Spring 96) and homework (47%, Fall 95, 83%, Winter 96, and 76%, Spring 96) increased throughout the quarter,
(ii) The workshops were a positive learning environment (an average of 91% agreed to strongly agreed throughout the year); and
(iii) The lab projects were a useful course component overall (an average of 69% consistently rated them as useful during the year).

The Initiative's new approach to calculus has integrated both content and instructional changes that emphasize active learning. This preliminary evaluation suggests that these approaches are changing student outcomes in calculus at Minnesota and, more importantly, influencing student understanding of how to use calculus as a tool for problem-solving across other disciplines.

Note: Further details are provided in a forthcoming report, which is available from the authors. Additional information can also be found in [1].

## References

[1] J. Leitzel, editors, *Assessing calculus reform efforts*, MAA, 1995.

[2] H. Keynes and A. Olson, "Calculus reform as a lever for changing curriculum and instruction", pp. 248–251 in *Proceedings Fourth World Conference on Engineering Education*, St. Paul, MN, 1995.

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