

## The Convex Geometry and Geometric Analysis Program MSRI, Spring 1996

During the last ten years the integral geometry of convex bodies has undergone a dramatic revitalisation, brought about by the introduction of methods, results and, most importantly, new viewpoints, from probability theory, harmonic analysis and the geometry of finite-dimensional normed spaces. The principal goal of this program was to bring together researchers from several different fields, Classical Convex Geometry, Geometric Functional Analysis, Computational Geometry and related areas of Harmonic Analysis. The main reason for doing so was that research in these areas has found considerable overlap in recent years. Several problems and classes of problems have been come upon independently from different directions, and techniques from some areas have been found important in others. This goal was achieved beyond even our most optimistic expectations.

As well as an introductory workshop, consisting of four lecture series with an educational format, the program included one full-scale research workshop and two concentrations of visitors, in addition to the regular activity during the principal five months. About 190 mathematicians attended the program in some capacity or other and there were over 150 lectures and seminars during the period. These were of several types. There was a regular educational seminar, two or three times a week which enabled participants to become acquainted with material from other fields. Three or four lectures a week dealt with recent research by members, and there was a “young research seminar” (roughly once a week) which gave postdocs and students a chance to describe their work in an informal atmosphere.

MSRI provided an unique environment in which to bring together such a group of people. The spectacular scenery served as a fitting backdrop to the immensely invigorating mathematical atmosphere.

The articles in this collection present recent research in the areas covered by the program. All this research was either completed at MSRI or presented in

lectures during the program. The full list of lectures is included in the next few pages, in order to give some indication of the scope of the program. Since this volume takes the place of the regular GAFA seminar series for the year 1995–96, we have also included a list of lectures given in that seminar.

We would like to express our sincere thanks to Silvio Levy for his careful preparation of the manuscript of these proceedings.

Keith Ball  
Vitali Milman

## MSRI PROGRAM SEMINARS

### Introductory Workshop on Convex Geometry and Geometric Functional Analysis

The contents of this introductory workshop were published in the form of two sections, by Ball and Bollobás respectively, in the book *Flavors of Geometry*, Silvio Levy (editor), MSRI Publications **31**, Cambridge University Press, 1997.

*January 29, 1996*

K. Ball, Basic notions in convex geometry

*January 30, 1996*

K. Ball, Fritz John's theorem

G. Schechtman, The central limit theorem and large deviation inequalities

*January 31, 1996*

J. Lindenstrauss, Volume ratios and their uses

*February 1, 1996*

K. Ball, The Brunn–Minkowski theorem

G. Schechtman, Concentration of measure in geometry

*February 2, 1996*

J. Lindenstrauss, Embedding Euclidean spaces into  $l_1$

*February 5, 1996*

B. Bollobás, Rapid mixing and volume estimation, part I

K. Ball, Convolutions and volume ratios

*February 6, 1996*

G. Schechtman, Dvoretzky's theorem

*February 7, 1996*

B. Bollobás, Rapid mixing and volume estimation, part II

K. Ball, The slicing problem (Informal talk)

*February 8, 1996*

J. Lindenstrauss, Distributing points uniformly on spheres

*February 9, 1996*

B. Bollobás, Rapid mixing and volume estimation, part III

### Regular Seminars

*January 17, 1996*

K. Ball, A new lower bound for lattice packing density in high dimensions

*February 2, 1996*

S. Kwapien, An inductive approach to moment estimates on product spaces

*February 5, 1996*

K. Ball, Volume in  $\mathbb{R}^n$  and its relationship to linear structure (MSRI–UC Berkeley Lecture)

*February 6, 1996*

A. Dembo, Information inequalities and concentration of measure

*February 7, 1996*

S. Kwapien, An inductive approach to moment inequalities, part II

*February 8, 1996*

S. Dar, The isotropic constant of non-symmetric convex bodies

*February 12, 1996*

D. Klain, Valuations and Hadwiger characterization theorem

G. Schechtman, Alexandrov–Fenchel inequalities, part I

*February 13, 1996*

S. Szarek, A ‘restricted’ Brunn–Minkowski inequality

A. Giannopoulos, On some vector balancing problems

*February 14, 1996*

G. Schechtman, Alexandrov–Fenchel inequalities, part II

*February 15, 1996*

J. Mount, Sampling contingency tables

N Tomczak–Jaegermann, Complexity and higher order Schreier families

*February 16, 1996*

S. Alesker, Hilbert polynomial and number of points in the sum of finite sets (after Khovanskii)

### **Concentration in Infinite Dimensional Convex Geometry**

*February 20, 1996*

N. Kalton, Complements of Sidon sets

E. Odell, Proximity to  $l_1$  and distortion in asymptotic  $l_1$  spaces

J. Lindenstrauss, The uniform classification of Banach spaces

B. Randriantoanina, 1-complemented subspaces in complex sequence spaces

N. Randriantoanina, Absolutely summing operators on non-commutative  $C^*$ -algebras

M. Girardi, Completely continuous operators

*February 21, 1996*

P. Wojtaszczyk, Wavelets for Banach spaces

T. Gamelin, Hankel operators on bounded analytic functions

H. Rosenthal, Invariants for differences of bounded semi-continuous functions, with applications to Banach space theory

S. Dilworth, Differentiability of the Pettis integral and weak and scalar convergence almost everywhere

A. Koldobsky, The Levy representation of norms and inequalities for Gaussian expectations

D. Speegle, A construction of indicator function wavelets

P. Habala, A Banach space whose subspaces do not have the GL-property

*February 22, 1996*

M. Ostrovskii, Classes of Banach spaces stable with respect to the opening

V. Fonf, Countable proximal sets in infinite dimensional Banach spaces

S. Argyros, Convex unconditionality and summability of weakly null sequences

T. Schlumprecht, A Banach space with hereditarily huge asymptotic structure

R. Wagner, Gowers dichotomy for asymptotic structure

G. Androulakis, Distorting mixed Tsirelson spaces

R. Judd, Calculating the  $l_1$  index of certain Banach spaces

*February 23, 1996*

B. Maurey, Banach spaces with small spaces of operators

G. Godefroy, Progress on the approximation properties

P. Casazza, Complemented unconditional basic sequences in Banach lattices

I. Deliyanni, Examples of asymptotic  $l_1$  Banach spaces

D. Kutzarova, On some asymptotic  $l_1$  spaces

M. Robdera, On the analytic complete continuity property

P. Saab, On convolution operators associated with vector measures

S. Saccone, Tight uniform algebras

## Regular Seminars

*February 26, 1996*

J. Lindenstrauss, Zonoids whose polars are zonoids

*February 27, 1996*

M. Wodzicki, A fresh look at Banach spaces

M. Girardi, Strongly measurable Banach-space valued functions: examples and results

*February 28, 1996*

M. Rudelson, Direct construction of majorizing measures

*February 29, 1996*

J. M. Rojas, Affine space and toric varieties, part II

*March 4, 1996*

Y. Gordon, Volume computation for quotients of  $L_p$  spaces and applications

*March 5, 1996*

R. Schneider, From areas in Minkowski spaces to zonoids

K. Swanepoel, Collapsing conditions for sets of vectors in Minkowski spaces

*March 6, 1996*

- A. Arias, Pisier's example of a polynomially bounded operator which is not similar to a contraction (after G. Pisier)
- A. Petrunin, Introduction to Alexandrov spaces

*March 8, 1996*

- V. Milman, Some problems in local theory
- E. Grinberg, Microlocal analysis of convex surfaces and Funk's characterization of the sphere

### **Workshop in Random Methods in Convex Geometry**

*March 11, 1996*

- V. Milman, Global versus local views in high dimensional convexity
- M. Talagrand, A functional point of view for concentration of measure
- A. Zee, Universal correlation and other results in random matrix theory
- B. Bollobás, Random partial orders
- S. Szarek, Komlos conjecture, Sidak's inequality and local Lovász lemma
- S. Alesker, Integrals of analytic and smooth functions over Minkowski sum of convex bodies
- R. Latała, Estimates of moments of sums of independent real random variables

*March 12, 1996*

- R. Kannan, Logarithmic Sobolev inequalities and geometric random walks
- M. Simonovits, Randomized volume algorithms
- H. König, Isometric embeddings of Euclidean spaces into  $l_p^N$  spaces and cubature formulas on spheres
- M. Junge, Mixed volumes for  $l_p$  sums of convex bodies
- A. Tsolomitis, Limiting convolution bodies
- M. Schmückenschlager, Hu's inequality
- J. Wenzel, Sequences of ideal norms and the UMD-property

*March 13, 1996*

- R. Schneider, Determination of convex bodies from projection functions
- J. Pach, On uniformly distributed distances — a geometric application of Janson's inequality
- M. Rudelson, Contact points and applications
- B. Bollobás, Volume estimates and rapid mixing (MSRI–UC Berkeley Lecture)

*March 14, 1996*

- I. Barany, Affine perimeter and limit shape
- W. Banaszczyk, The width of lattice-point-free convex bodies
- P. Mankiewicz, Groups of operators acting on random quotients of  $l_1^m$
- A. Giannopoulos, Low  $M^*$  estimates for coordinate subspaces
- T. Schlumprecht, The Gaussian correlation problem for ellipsoids
- S. Dar, Slicing problem for trace classes

*March 15, 1996*

- D. Welsh, Randomized approximation of geometrical Tutte invariants
- S. Kwapien, Differential inequalities and comparison of moments
- F. Chung, Logarithmic Harnack inequalities
- G. Zhang, Ellipsoidal decompositions of centered bodies
- W. Weil, Section and projection means of convex bodies
- D. Klain, A continuous analogue of Sperner's theorem
- R. Vitale, The Wills functional and Gaussian processes

### Regular Seminars

*March 18, 1996*

- R. Vitale, The Wills functional and Gaussian processes II

*March 19, 1996*

- A. Tsolomitis, Convolution bodies
- F. Barthe, Extremal sections of the unit ball of  $l_p^n$

*March 20, 1996*

- V. Klee, Some unsolved problems in convex geometry: seven favorites
- W. Banaszczyk, Some inequalities for polar reciprocal  $n$ -dimensional lattices and convex bodies (transference theorems in the geometry of numbers)

*March 22, 1996*

- W. Weil, Local formulae in integral geometry

*March 25, 1996*

- D. Klain, Blaschke sums and mixed bodies,

*March 26, 1996*

- N. Tomczak-Jaegermann, Spaces of type  $p$  containing arbitrarily distortable subspaces,

*March 27, 1996*

- E. Lutwak, The Brunn–Minkowski–Firey theory,

*March 28, 1996*

- G. Zhang, Volume inequalities for sections of convex bodies,

*March 29, 1996*

- I. Bárány, On the number of convex lattice polytopes,

*April 1, 1996*

- M. Simonovits, Localization lemmas and isoperimetric inequalities, part I

*April 2, 1996*

- K. M. Ball, Bang's Lemma and the De Leeuw–Kahane–Katznelson Theorem (d'après Nazarov)
- V. Ferenczi, Several properties of hereditarily indecomposable Banach spaces

*April 3, 1996*

A. Pajor, Kolmogorov's entropy in convex geometry

*April 4, 1996*

W. B. Johnson, Extensions of  $c_o$ : an addendum to a paper by Kalton and Pelczynski

*April 5, 1996*

M. Simonovits, Localization lemmas and isoperimetric inequalities, part II

A. Petrunin, Alexandrov spaces, part II

*April 9, 1996*

A. Petrunin, Alexandrov spaces, part III

P. Habala, A Banach space whose subspaces fail the Gordon–Lewis property

*April 1, 1996*

B. Maurey, Factorization of linear operators

*April 12, 1996*

W. T. Gowers, A lower bound of tower type for Szemerédi's uniformity lemma

A. Khovanskii, Connections between algebraic and convex geometry

*April 15, 1996*

A. Khovanskii, Connections between algebraic and convex geometry, part II

*April 16, 1996*

N. Ghoussoub, Improved Moser–Aubin–Onofri's inequalities on  $S^2$

R. Latała, On the equivalence between arithmetic and geometric mean for logarithmically concave measures

*April 17, 1996*

J. M. Rojas, Mixed subdivisions and some practical results on mixed volume computation

S. J. Szarek, Free probability, part I

*April 19, 1996*

S. J. Szarek, Free probability, part II

*April 22, 1996*

P. M. Gruber, Some aspects of approximation of convex bodies by polytopes

*April 24, 1996*

G. Pisier, Quadratic forms in unitary operators

*April 26, 1996*

M. Loss, The Best Constant in the Hardy–Littlewood–Sobolev Inequality

*April 29, 1996*

S. Szarek, Free probability and random matrices

*April 30, 1996*

K. Oleszkiewicz, On the discrete version of the antipodal theorem

S. Dar, A Brunn–Minkowski type inequality



*May 1, 1996*

J. Bourgain, Influence of variables and threshold intervals

*May 2, 1996*

E. Carlen, Logarithmic Sobolev inequalities and sharp estimates for smoothing and decay for the 2-d Navier–Stokes equation

*May 3, 1996*

J. Bourgain, Random points in isotropic convex sets

T. Oikhberg, Exact operator spaces with exact dual

G. Pisier, Similarity problems and completely bounded maps

*May 6, 1996*

S. Bates, Nonlinear surjections of Banach spaces

*May 7, 1996*

A. Pelczynski, The Dunford–Pettis property of  $L_1$  on cosidon sets

### **Sharp Inequalities in Harmonic Analysis and Convex Geometry**

*May 8, 1996*

K. Ball, Convolution inequalities and convex geometry

K. Oleszkiewicz, Best constant in the Khinchine–Kahane inequality

B. Ruskai, Contraction of relative entropy in information theory and quantum theory

*May 9, 1996*

G. Pisier, Various inequalities for vector valued  $C_p$  spaces

R. Ambartzumian, Hilbert’s fourth problem: parametric versions

C. Morpurgo, Zeta functions and fractional integral inequalities

E. Grinberg, The cosine transform of higher order

*May 10, 1996*

D. Burkholder, Some sharp inequalities in stochastic analysis

A. Barvinok, Computing mixed discriminant, mixed volumes, and permanents

D. Jerison, Variational problems for the capacity and for the first eigenvalue of convex bodies

*May 13, 1996*

A. Burchard, The Riesz rearrangement inequality

C. Borell, On Brunn–Minkowski inequalities in option theory

T. Bisztriczky, A proof of Hadwiger’s conjecture for dual cyclic polytopes

S. Alesker, Polynomial rotation invariant valuations on convex sets

*May 14, 1996*

E. Lieb, Inequalities related to stability of matter: overview

R. Laugesen, Extremals for zeta functions (and more) on Laplacians under conformal mapping

*May 15, 1996*

M. Loss, Inequalities related to stability of matter: matter and fields

### **Regular Seminars**

*May 14, 1996*

A. Litvak, An extension of reverse Brunn–Minkowski inequality to non-convex case

*May 16, 1996*

R. Wagner, Asymptotic versions of operators and operator ideals

*May 17, 1996*

E. Lieb, Gaussian kernels and some of their applications

*May 20, 1996*

G. Schechtman, Three problems in geometric functional analysis

*May 22, 1996*

L. Lovasz, Stopping rules for random walks in convex bodies

*May 23, 1996*

L. Tzafrizi, Legendre and Jacobi polynomials in Banach Space theory

*May 24, 1996*

A. Giannopoulos, On the diameter of proportional sections of a symmetric convex body

*May 28, 1996*

M. Rudelson, Bringing a body into an isotropic position

*May 29, 1996*

E. Gluskin, On Kashin approach to correction theorems

M. Loss, On the paper of Elliott Lieb — Gaussian kernels have Gaussian optimizers

*May 30, 1996*

H. Groemer, Half-sections and half-girths of convex bodies

## GAGA SEMINARS 1994–1996

*November 25, 1994*

- Yaki Sternfeld (Haifa), Recent problems and methods in dimensional theory  
M. Dubiner (Tel Aviv), The equivalence of two polynomially defined metrics on a general convex subset of  $\mathbb{R}^n$

*December 9, 1994*

- M. Krivelevich (Tel Aviv), Probabilistic techniques in Ramsey theory  
S. Reisner (Haifa), Constructing a polytope to approximate a convex body (joint work with Y. Gordon (Haifa) and Y. Meyer (France))  
M. Rudelson (Jerusalem), Approximate John's decomposition

*December 23, 1994*

- S. Dar (Tel Aviv), Isotropic capacity of convex non-symmetric bodies  
Joel Zinn (Texas), Hypercontractivity and a Gaussian correlation inequality  
A. Leiderman (Beer Sheva), The Kolmogorov superposition theorem and the free locally convex space on the unit interval

*April 7, 1995*

- G. Schechtman (Rehovot), Isomorphic version of Dvoretzky's theorem (joint work with V. Milman (Tel Aviv))  
G. Schechtman (Rehovot), A peculiar rearrangement of the Haar system (joint work with P. Muller)  
R. Wagner (Tel Aviv), Asymptotic constants of Tzirelson type spaces (joint work with E. Odell and N. Tomczak)

*November 24, 1995*

- V. Milman (Tel Aviv), Global vs. local results in asymptotic theory of normed spaces (joint work with G. Schechtman (Rehovot))  
H. Hofer (Zurich), From periodic orbits to symplectic homology  
A. Shnirelman (Tel Aviv), Geometry and dynamics on the group of area-preserving diffeomorphisms

*December 8, 1995*

- S. Dar (Tel Aviv), Gradient map and Legendre transform (after M. Gromov)  
S. Alesker (Tel Aviv), Minkowski type theorems for smooth and analytical weights  
J. Lindenstrauss (Jerusalem), Differentiability of Lipschitz maps between Banach spaces

*December 29, 1995*

- I. Benjamini (Rehovot), On the support of harmonic measures for random walks  
Y. Gordon (Haifa), Variants of the Cauchy–Binet formula, with applications to volume estimates (joint work with M. Junge)  
M. Braverman (Beer Sheva), Rosenthal's inequality and characterization of  $L_p$ -spaces

*January 12, 1996*

- Y. Benjamini (Haifa), Linear approximation of quasi-isometries on  $\mathbb{R}^n$  (work of Fritz John)
- A. Dembo (Haifa), Information inequalities and concentration of measure