STOCHASTIC QUANTIZATION

Summer Graduate School, MSRI/SLMath
July 1 to 12, 2024

Recommended Readings

Linear theory of Laplace equation and heat equation

[1] L. C. Evans, *Partial differential equations*, 2nd ed., Graduate Studies in Mathematics **19**, American Mathematical Society, Providence, RI, 2010, 3rd printing of 2nd ed corrects all errors as of February 2022. ISBN 978-0-8218-4974-3. MR 2597943. Zbl 1194.35001. doi: 10.1090/gsm/019. Sections 2.2 and 2.3.

Finite dimensional Gaussian measures

[2] B. Øksendal, *Stochastic differential equations: An introduction with applications*, 6th ed., Universitext, Springer, Cham, Switzerland, 2013, 6th corrected printing, ISBN 3-540-04758-1. MR 2001996. Zbl 1025.60026. doi: 10.1007/978-3-642-14394-6. Appendix A.

Infinite dimensional Gaussian measures. White noise.

[3] G. Da Prato, *An introduction to infinite-dimensional analysis*, 2nd ed., Universitext, Springer, Cham, Switzerland, 2006, ISBN 978-3-540-29020-9; 3-540-29020-6. MR 2244975. Zbl 1109.46001. doi: 10.1007/3-540-29021-4. Chapter 1.

Brownian motion. Stochastic differential equations with additive Brownian noise.

[4] B. Øksendal, *Stochastic differential equations: An introduction with applications*, 6th ed., Universitext, Springer, Cham, Switzerland, 2013, 6th corrected printing, ISBN 3-540-04758-1. MR 2001996. Zbl 1025.60026. doi: 10.1007/978-3-642-14394-6. Sections 2, 3, 4, 5.

Schwarz distributions. Fourier transform on a discrete lattice and on Euclidean space.

[5] L. C. Evans, *Partial differential equations*, 2nd ed., Graduate Studies in Mathematics **19**, American Mathematical Society, Providence, RI, 2010, 3rd printing of 2nd ed corrects all errors as of February 2022. ISBN 978-0-8218-4974-3. MR 2597943. Zbl 1194.35001. doi: 10.1090/gsm/019. Section 4.3.1.

[6] W. Rudin, *Functional analysis*, 2nd ed., International Series in Pure and Applied Mathematics, McGraw-Hill, New York, NY, 1991, ISBN 0-07-054236-8. MR 1157815. Zbl 0867.46001. Sections 6 and 7.

Weak convergence of probability measures. Tightness of probability measures.

[7] R. Durrett, *Probability: theory and examples*, 5th ed., Cambridge Series in Statistical and Probabilistic Mathematics **49**, Cambridge University Press, Cambridge, UK, 2019, ISBN 978-1-108-47368-2. MR 3930614. Zbl 1440.60001. doi: 10.1017/9781108591034. Sections 3.2 and 3.3.

Date: May 10, 2024.
Bibliography by Hao Shen.

Books on Analysis

[8] R. Durrett, *Probability: theory and examples*, 5th ed., Cambridge Series in Statistical and Probabilistic Mathematics **49**, Cambridge University Press, Cambridge, UK, 2019, ISBN 978-1-108-47368-2. MR 3930614. Zbl 1440.60001. doi: 10.1017/9781108591034.

[9] L. C. Evans, *Partial differential equations*, 2nd ed., Graduate Studies in Mathematics **19**, American Mathematical Society, Providence, RI, 2010, 3rd printing of 2nd ed corrects all errors as of February 2022. ISBN 978-0-8218-4974-3. MR 2597943. Zbl 1194.35001. doi: 10.1090/gsm/019.

[10] W. Rudin, *Functional analysis*, 2nd ed., International Series in Pure and Applied Mathematics, McGraw-Hill, New York, NY, 1991, ISBN 0-07-054236-8. MR 1157815. Zbl 0867.46001.

Books on Probability

[11] P. Billingsley, *Convergence of probability measures*, 2nd ed., Wiley Series in Probability and Statistics, Wiley, New York, NY, 1999, ISBN 0-471-19745-9. MR 1700749. Zbl 0944.60003. doi: 10.1002/9780470316962.

[12] G. Da Prato, *An introduction to infinite-dimensional analysis*, 2nd ed., Universitext, Springer, Cham, Switzerland, 2006, ISBN 978-3-540-29020-9; 3-540-29020-6. MR 2244975. Zbl 1109.46001. doi: 10.1007/3-540-29021-4.

[13] G. Da Prato and J. Zabczyk, *Stochastic equations in infinite dimensions*, 2nd ed., Encyclopedia of Mathematics and its Applications **152**, Cambridge University Press, Cambridge, UK, 2014, ISBN 978-1-107-05584-1. MR 3236753. Zbl 1317.60077. doi: 10.1017/CBO9781107295513.

[14] P. K. Friz and M. Hairer, *A course on rough paths: with an introduction to regularity structures*, 2nd ed., Universitext, Springer, Cham, Switzerland, 2020, ISBN 978-3-030-41556-3; 978-3-030-41555-6. MR 4174393. Zbl 1437.60002. doi: 10.1007/978-3-030-41556-3.

[15] D. Nualart and E. Nualart, *Introduction to Malliavin calculus*, Institute of Mathematical Statistics Textbooks **9**, Cambridge University Press, Cambridge, UK, 2018, ISBN 978-1-107-61198-6; 978-1-107-03912-4. MR 3838464. Zbl 1425.60002. doi: 10.1017/9781139856485.

[16] B. Øksendal, *Stochastic differential equations: An introduction with applications*, 6th ed., Universitext, Springer, Cham, Switzerland, 2013, 6th corrected printing, ISBN 3-540-04758-1. MR 2001996. Zbl 1025.60026. doi: 10.1007/978-3-642-14394-6.

Books on Mathematical Physics

[17] S. Friedli and Y. Velenik, *Statistical mechanics of lattice systems: a concrete mathematical introduction*, Cambridge University Press, Cambridge, UK, 2018. MR 3752129. Zbl 1407.82001. doi: 10.1017/9781316882603.

[18] J. Glimm and A. Jaffe, *Quantum physics: a functional integral point of view*, 2nd ed., Springer, Cham, Switzerland, 1987, ISBN 0-387-96476-2. MR 887102. Zbl 0461.46051. doi: 10.1007/978-1-4612-4728-9.

[19] B. C. Hall, *Quantum theory for mathematicians*, Graduate Texts in Mathematics **267**, Springer, Cham, Switzerland, 2013. MR 3112817. Zbl 1273.81001. doi: 10.1007/978-1-4614-7116-5.

[20] B. Simon, *The $P(\phi)_2$ Euclidean (quantum) field theory*, Princeton Series in Physics, Princeton University Press, Princeton, NJ, 1974. MR 489552. Zbl 1175.81146. https://press.princeton.edu/books/ebook/9781400868759/p02-euclidean-quantum-field-theory.

[21] F. Strocchi, *An introduction to the mathematical structure of quantum mechanics: a short course for mathematicians*, 2nd ed., Advanced Series in Mathematical Physics **28**, World Scientific, Singapore, 2008. MR 2484367. Zbl 1155.81004. doi: 10.1142/7038.

[22] L. A. Takhtajan, *Quantum mechanics for mathematicians*, Graduate Studies in Mathematics **95**, American Mathematical Society, Providence, RI, 2008. MR 2433906. Zbl 1156.81004. doi: 10.1090/gsm/095.

Some Relevant Papers

[23] N. Barashkov and M. Gubinelli, A variational method for Φ^4_3, *Duke Math. J.* **169** (2020), no. 17, 3339–3415. MR 4173157. Zbl 1508.81928. doi: 10.1215/00127094-2020-0029.

[24] Y. Bruned, F. Gabriel, M. Hairer, and L. Zambotti, Geometric stochastic heat equations, *J. Amer. Math. Soc.* **35** (2022), no. 1, 1–80. MR 4322389. Zbl 1476.60101. doi: 10.1090/jams/977.

[25] Y. Bruned, M. Hairer, and L. Zambotti, Algebraic renormalisation of regularity structures, *Invent. Math.* **215** (2019), no. 3, 1039–1156. MR 3935036. Zbl 1481.16038. doi: 10.1007/s00222-018-0841-x.

[26] Y. Bruned, M. Hairer, and L. Zambotti, Renormalisation of stochastic partial differential equations, *Eur. Math. Soc. Newsl.* (2020), no. 115, 7–11. MR 4226805. Zbl 1447.35399. doi: 10.4171/news/115/3.

[27] A. Chandra, I. Chevyrev, M. Hairer, and H. Shen, Langevin dynamic for the 2D Yang–Mills measure, *Publ. Math. Inst. Hautes Études Sci.* **136** (2022), 1–147. MR 4517645. Zbl 1518.70029. doi: 10.1007/s10240-022-00132-0.

[28] M. Gubinelli and M. Hofmanová, Global solutions to elliptic and parabolic Φ^4 models in Euclidean space, *Comm. Math. Phys.* **368** (2019), no. 3, 1201–1266. MR 3951704. Zbl 1420.35481. doi: 10.1007/s00220-019-03398-4.

[29] M. Gubinelli and M. Hofmanová, A PDE construction of the Euclidean ϕ^4_3 quantum field theory, *Comm. Math. Phys.* **384** (2021), no. 1, 1–75. MR 4252872. Zbl 1514.81190. doi: 10.1007/s00220-021-04022-0.

[30] M. Gubinelli, P. Imkeller, and N. Perkowski, Paracontrolled distributions and singular PDEs, *Forum Math. Pi* **3** (2015), Article ID e6. MR 3406823. Zbl 1333.60149. doi: 10.1017/fmp.2015.2.

[31] M. Hairer, A theory of regularity structures, *Invent. Math.* **198** (2014), no. 2, 269–504. MR 3274562. Zbl 1332.60093. doi: 10.1007/s00222-014-0505-4.

[32] M. Hairer and H. Shen, The dynamical sine-Gordon model, *Comm. Math. Phys.* **341** (2016), no. 3, 933–989. MR 3452276. Zbl 1336.60120. doi: 10.1007/s00220-015-2525-3.

[33] H. Shen, R. Zhu, and X. Zhu, A stochastic analysis approach to lattice Yang–Mills at strong coupling, *Comm. Math. Phys.* **400** (2023), no. 2, 805–851. MR 4589716. Zbl 1522.81127. doi: 10.1007/s00220-022-04609-1.