

# INTRODUCTION TO THE THEORY OF ALGEBRAIC CURVES

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University of California-Berkeley

[1] **The topics**

- (1) **moduli spaces of stable curves**
- (2) **Brill–Noether theory**
- (3) **the extrinsic geometry of the curves in projective space**

can be found in chapters V and VI of:

R. MIRANDA, *Algebraic curves and Riemann surfaces*, Graduate Studies in Mathematics **5**, American Mathematical Society, Providence, RI, 1995, ISBN 0-8218-0268-2. MR 1326604. Zbl 0820.14022. doi: 10.1090/gsm/005.

Specifically, to help you get started:

- Chapter I has the definition of a Riemann surface (which is the same thing as an algebraic curve over the complex numbers) and some nice first examples.
- Chapter II discusses holomorphic and meromorphic functions on Riemann surfaces. Section 4 of this chapter contains the Riemann–Hurwitz formula.
- You can check the definition of a holomorphic/meromorphic 1-form at the start of chapter IV, and then proceed to chapter V, which discusses divisors and maps to projective spaces.
- Chapter VI contains the Riemann–Roch theorem.

[2] **Some alternate sources for more advanced students are chapters 15, 18, 19, and 21 of:**

R. VAKIL, *The rising sea: foundations of algebraic geometry*.  
<http://www.freetechbooks.com/the-rising-sea-foundations-of-algebraic-geometry-t1382.html>.  
Open access.

[3] **or chapter IV of:**

R. HARTSHORNE, *Algebraic geometry*, Graduate Texts in Mathematics **52**, Springer, Cham, Switzerland, 1977, ISBN 0-387-90244-9. MR 463157. Zbl 0367.14001. doi: 10.1007/978-1-4757-3849-0.4.